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Bottom-up Default Analysis (BuDA v3.3.1)

White Paper

The Credit Research Initiative (CRI)
National University of Singapore

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Credit Research Initiative
National University of Singapore

ABSTRACT

Bottom-up Default Analysis (BuDA) is a credit stress testing and scenario analysis toolkit jointly developed by the Credit Research Initiative (CRI) team of the National University of Singapore (NUS) and the International Monetary Fund (IMF), and operationally linked to the CRI platform (https://www.nuscri.org). BuDA provides a unique framework to evaluate the probabilities of default (PDs) of individual firms under prescribed macroeconomic scenarios, which are in turn used to aggregate bottom-up to a portfolio-level credit assessment. This toolkit, for example, enables IMF economists to anticipate consequences of envisioned macroeconomic/financial circumstances on each and/or a group of economies/industries, which is fundamental to policymaking. This white paper provides the conceptual background underpinning BuDA. It offers a basic understanding of the inner workings of the BuDA toolkit as well as the various methods deployed to produce the individual and portfolio PDs under a scenario.

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BuDA (v1.0) was developed by Jin-Chuan Duan of the NUS-CRI team and Weimin Miao of CriAT, a former NUS-CRI team member, in collaboration with Jorge Chan-Lau of IMF. The NUS-CRI team provides the continual development and support of the BuDA platform.

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I. OVERVIEW

BuDA is a bottom-up approach to credit stress testing and scenario analysis. The key question within the analysis is, "How does the credit quality of an economy/sector (or a group of economies/sectors) respond to shocks to macroeconomic variables/common risk drivers?" For example, under a prolonged recession with consecutive quarters of negative GDP growth, how badly would the economy/sector's credit profile deteriorate?

The basic elements are individual firm-level PDs of listed firms which, collectively, make up the corporate sector of an economy. CRI produces PDs for virtually all exchange-listed firms in the world, and the work of BuDA is to translate the impact of macroeconomic shocks on individual firm PDs, which are then aggregated bottom-up to any target economy/sector.

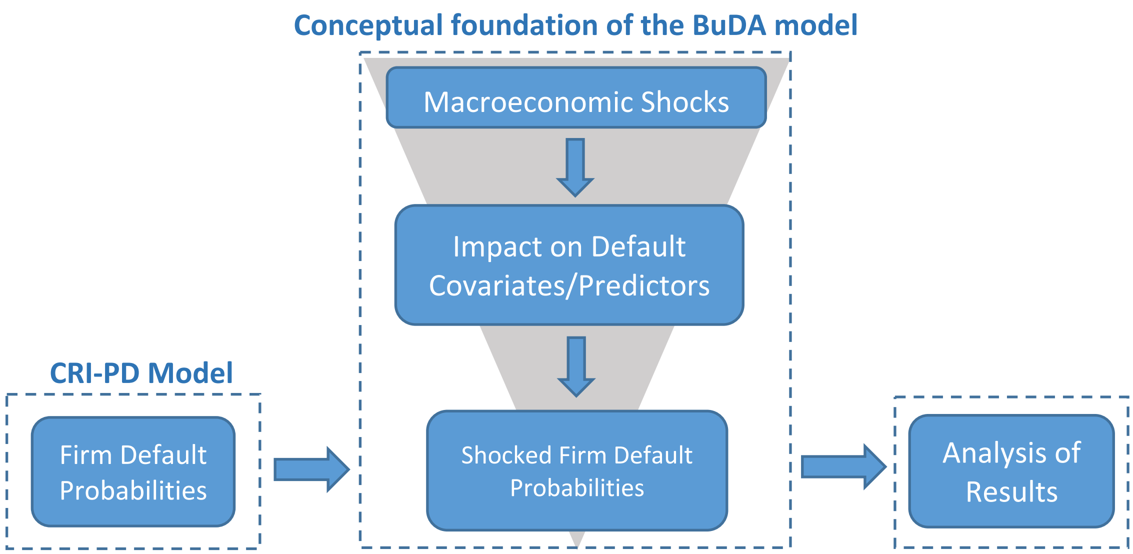

Figure 1: Conceptual overview of the BuDA Framework

II. METHODOLOGY

The BuDA framework comprises two main components. Firstly, we define default events and describe the CRI-PD model. Secondly, we explain the methodological foundation underlying the BuDA approach to macroeconomic scenario stress-testing analysis.

CRI-PD

What are probabilities of default (PD)?

When money changes hands, the lender is concerned about whether debt repayments, including interest and principal payments, will take place as originally agreed upon with the borrower. The failure of the latter to deliver on its repayment obligations constitutes a default event. The default events recognized by CRI can be classified under one of the following events:

1. Bankruptcy filing, receivership, administration, liquidation or any other legal impasse to the timely settlement of interest and/or principal payments;
2. A missed or delayed payment of interest and/or principal, excluding delayed payments made within a grace period;
3. Debt restructuring/distressed exchange, in which debt holders are offered a new security or package of securities that result in a diminished financial obligation (e.g., a conversion of debt to equity, debt with lower coupon or par value, debt with lower seniority, debt with longer maturity).

In the CRI operational framework, default events fall under at least one of three categories: (1) a bankruptcy filing, (2) a delisting followed by bankruptcy filing, or (3) a default corporate action. Table 1 and Table 2 list firm exits which are considered a default and other form of corporate exit, respectively. These default events are consistent with those adopted by major rating agencies and encompass similar categories of events despite differences in the legal framework across countries.

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Table 1: Firm Exits Classified as Defaults

Default	
Action Type	**Subcategory**
Bankruptcy filing	Administration, Arrangement, Canadian Companies' Creditors Arrangement Act (CCAA), Chapter 7,11,15 (United States bankruptcy code), Conservatorship, Insolvency, Japanese Corporate Reogranization Law (CRL), Judicial management, Liquidation, Pre-negotiation Chapter 11, Protection, Receivership, Rehabilitation, Rehabilitation (Thailand 1997), Reorganization, Restructuring, Section 304, Supreme Court declaration, Winding up, Work out, Sued by creditor, Petition withdrawn
Delisting	Followed by bankruptcy
Default corporate action	Bankruptcy, Coupon & principal payment, Coupon payment only, Debt restructuring, Interest payment, Loan payment, Principal payment, Alternative Dispute Resolution (ADR, Japan only), Declared sick (India only), Regulatory action (Taiwan only), Financial difficulty and shutdown (Taiwan only), Buyback option

Table 2: Firm Exits Classified as "Other Exits"

Other Exits	
Action Type	**Subcategory**
Delisting	Acquired/merged, Assimilated with underlying shares, Bid price below minimum, Cancellation of listing, Failure to meet listing requirements, Failure to pay listing fees, Inactive security, Insufficient assets, Insufficient capital and surplus, Insufficient number of market makers, Issue postponed, Lack of market maker interest, Lack of public interest, Liquidated, Not current in required filings, NP/FP finished, Privatized, Reorganization, Security called for redemptions, the company's request, Scheme of arrangement, Selective capital reduction of the company, From exchange to Over-the-Counter (OTC), Privatised

A default event could trigger several important responses. Company assets and liabilities may be reorganized or liquidated, depending on circumstances. Due to the lack of sufficient assets to meet liabilities, the stock price will severely decline or face a complete wipe-out. Likewise, bonds and debt instruments written by the company may lose value if investors believe that the ability to honor the obligations is severely impaired. Credit default swaps, essentially insurance on the company's bonds, will be triggered for pay out if they are traded.

Since losses associated with defaults can be substantial, there is interest among lenders/credit investors, analysts, financial regulators, and systemic risk supervisors in knowing the likelihood of default for a firm or group of firms. The most natural default

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risk measure of a firm is its Probability of Default (PD). PD conveys a sense of the credit quality of a company, with a lower value suggesting a better credit quality. An aggregate (average or median) of PDs can convey an assessment on the credit quality of a group of companies. If an average is used, it can be either simple or value-weighted depending on the user's purpose. Fundamentally, the companies with better credit quality are those with a healthier balance sheet position, liquidity, profitability, and lower stock volatility.

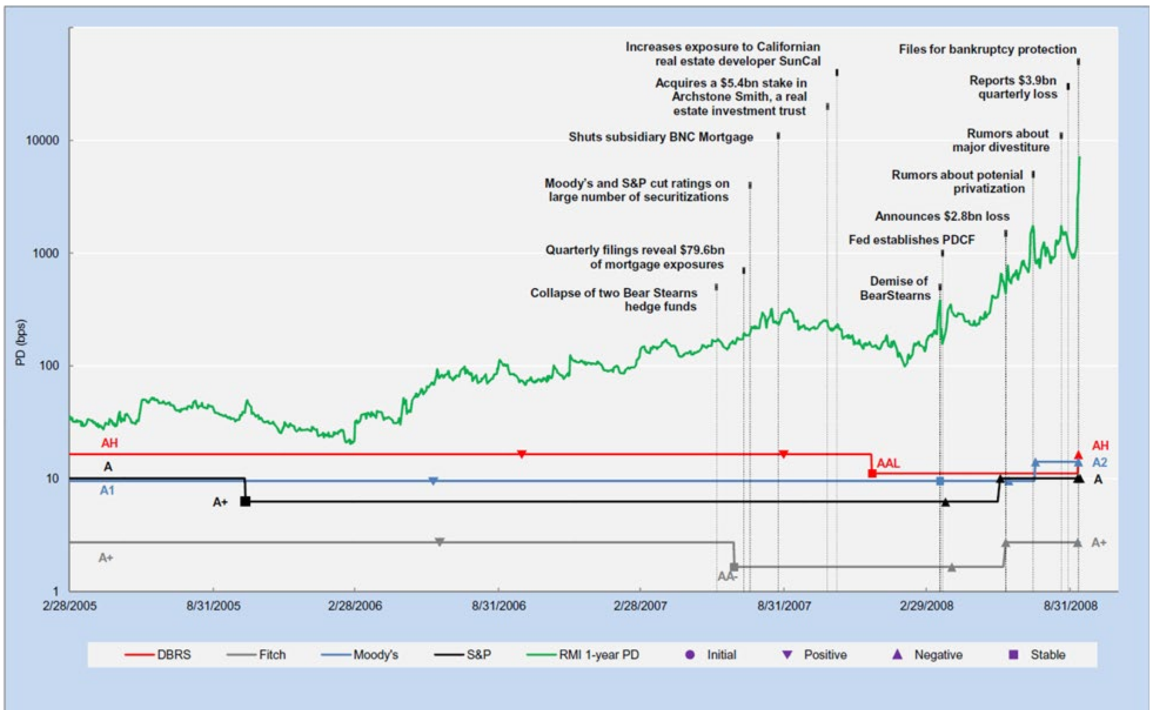


Figure 2: The Collapse of Lehman Brothers

*Figure obtained from "A Lead-Lag Investigation of RMI PD and CRA Ratings," Global Credit Review 2012

As an illustration, let us examine the saga concerning Lehman Brothers. Figure 2, taken from Global Credit Review, shows an analysis starting three and a half years prior to the company's September 2008 bankruptcy filing under Chapter 11 of the United States Bankruptcy Code. Through this period, the CRI 1-year PD rose steadily from less than 100 basis points (bps[1]) to almost 10,000 bps in September 2008. The vertical axis is in the log scale with each tick amounting to 10 times in PDs. On the same graph, the credit ratings over the same period issued by major credit rating agencies (CRAs) such as S&P, Moody's', etc. are plotted. The letter ratings offered by the CRAs have been converted into

[1] 100 basis points = 1%.

probabilities using their respective reported historical realized annual default rates. The graph clearly demonstrates that Lehman Brothers' risk of default was, according to CRI-PD, much higher than the credit risk assessments offered by CRAs at that time.

PD reveals an assessment as to how likely it is for a firm to default over a given horizon. Naturally, the stakeholders should be concerned when a company's PD rises beyond its historical levels, or the level at which the company enters a heightened credit risk position. For policy makers and credit portfolio investors, the focus may go well beyond a single firm. When the perspective is beyond a single company, one will need to assess the current levels of the aggregate PDs for specific industrial sectors and/or the broader economy, and their potential behaviors in response to changing economic conditions. For example, contemplate an economic recession scenario where GDP falls continually. This may cause the PDs of many companies to rise together, increasing the aggregate PD levels in the economy and raise systemic risk concerns. The BuDA approach, described in detail later, can identify and analyze such situations, helping regulators, lenders, economists, and market observers analyze and manage credit risks more adequately.

In comparison to most other sources of credit risk information, CRI-PDs being updated daily are more responsive. The model used to generate CRI-PDs is constructed with a cutting-edge default econometric approach and implemented with modern data analytical tools. The input data to the model include stock prices, prevailing interest rate, and financial ratios, among other information. The CRI-PDs, model, methodology and implementation are fully disclosed on the CRI website (https://www.nuscri.org). The CRI-PD model's implementation has been continually refined over time, and the model is recalibrated monthly to capture potential changes in the model's parameters. Details concerning the amendments and improvements made to the model are released at the https://nuscri.org/en/technical_document/ where easy reference to the technical documents can be found.

The CRI-PD model

In this section, we cover the key features of the CRI-PD model which is based on the forward-intensity modeling approach of Duan et (2012) with the aim of conveying the intuition behind the model. Technical details about the model available in a later section can be skipped if users are only interested in running BuDA.

Data

Estimating PDs is particularly challenging in part because their historical values are not directly observable. What available to analysts is whether a particular firm defaulted at some point of time in the past, but overall such defaults happen rather infrequently. Since there is no direct measurement of PD, one can only hope to empirically estimate PDs by applying a quantitative model on a large sample of historical records on the default/survival status of firms along with their relevant and observable attributes.

Therefore, we begin by observing actual realized defaults and other corporate exits and identify the circumstances under which the corporate events occur. Variables which are associated with or have predictive properties of default outcomes are called *default covariates/predictors*. Empirical studies reported in the literature have identified several covariates which seem to be highly associated with the occurrence of defaults, and they are incorporated into the CRI-PD model. Table 3 provides a list of the covariates and a brief description. The covariates fall under two major categories, common risk factors[2], and firm-specific attributes. The first category, as the name indicates, includes covariates that tend to affect all firms in the economy/sector.

Table 3: Covariates Used in Default Prediction

Covariates/predictors	Brief Description
Common risk factors	
Stock index	Trailing 1-year return on the stock market. A poor stock market performance is generally associated with weaker firm performance and more frequent occurrences of firm default.
Short-term interest rate	Yield on 3-month government bills. Generally speaking, if borrowing costs are higher, firms may face more funding constraints and may be more likely to default in the short-term. In the longer horizon, a higher interest rate may indicate positive economic growth and hence lower solvency risk.
Aggregate DTD	• Financial aggregate DTD is median DTD of financial firms in each economy inclusive of those foreign financial firms whose primary stock exchange is in this economy.

[2] In Duan et al (2012), common risk factors are referred to as macroeconomic risk factors. However, we reserve this term for scenario analysis/stress testing variables which we will discuss later in BuDA's methodology section.

	• Non-Financial aggregate DTD is median DTD of non-financial firms in each economy inclusive of those foreign non-financial firms whose primary stock exchange is in this economy.

(Table 3 – continue)

Covariate/predictor	Brief Description
Firm-specific attributes	
Distance-to-default • Level • Trend	Volatility-adjusted leverage measure based on Merton (1974), which is the logarithm of the ratio between the market value of a firm's assets and its liabilities, scaled by the asset volatility. A smaller DTD increases the likelihood of default. CRI uses a modified DTD measure which we will discuss later.
Liquidity • Level • Trend	Logarithm of the ratio of a firm's sum of cash and short-term investments to its total assets for financial firms. Logarithm of the ratio of current asset to current liability for non-financial firms. Higher liquidity is more beneficial for the firm.
Profitability • Level • Trend	Ratio of each firm's net income to total assets. Higher profitability is more beneficial for the firm.
Relative size • Level • Trend	Logarithm of the ratio of each firm's market capitalization to the economy's median market capitalization over the past year. Loosely speaking, bigger firms tend to have fewer defaults, although we sometimes observe the opposite.
Relative market-to-book ratio	Ratio of each firm's pseudo market value (market capitalization plus total book value of liabilities) to its book value (total book value of assets) relative to that of the economy. It captures the mis-valuation or growth opportunity effect.
Idiosyncratic volatility	Variation of firm returns which cannot be attributed to the stock market index, using daily data from the past year. Firms with higher idiosyncratic volatility tend to have more variable cash flows and a higher chance of bankruptcy.

CRI aims to generate and distribute the credit risk assessments on every exchange-listed firms of every country/economy globally. Thus far, major efforts have been put into collecting data on covariates/predictors, defaults and other corporate exits on over 80,000 firms in 133 economies across all continents. Among these firms, over 40,000 firms are currently active, and on an ongoing basis, effort has been committed to monitoring these firms and collecting the relevant data. Market-based data such as stock prices and interest rates are updated daily, while data from financial statements are checked daily and updated once available. The main sources of data are Thomson Reuters Datastream and the Bloomberg Data License Back Office Product. This rich information set has been used to generate CRI-PDs and to power the BuDA toolkit.

Of the 133 economies that we cover, more than 80 of them have national stock exchanges, and for each of those with a national exchange, a specific, representative stock index and a short-term interest rate are chosen. For the remaining economies, CRI covers the companies which are domiciled in the economy but quoted on a foreign exchange, because those economies do not have a stock exchange.

Financial statements and market data for individual firms serve to provide the remaining firm-specific covariates/predictors, namely, Distance-to-Default (DTD), liquidity, profitability, relative size, relative market-to-book ratio, and idiosyncratic volatility. Calculating the value of the covariates/predictors is straightforward, except for DTD, for which we provide a more in-depth explanation later.

Level and trend variables

To increase the predictive accuracy of the PD model, it is beneficial to include trend value, in addition to level value, for some firm-specific attributes. The level covariate is computed as the one-year moving average of the measure, and the trend covariate is calculated as its current value of the measure minus the one-year moving average. Considering Figure 3 below, Firm 1's DTD has been falling over consecutive periods, while Firm 2 has been rising. Even though they both currently sit at the same point, the statistical trends suggest that in the next period, it is quite likely that the DTD of Firm 1 will fall below Firm 2. Duan et al (2012) found that incorporating trend into the model significantly improves its predictive power for short-term horizons.

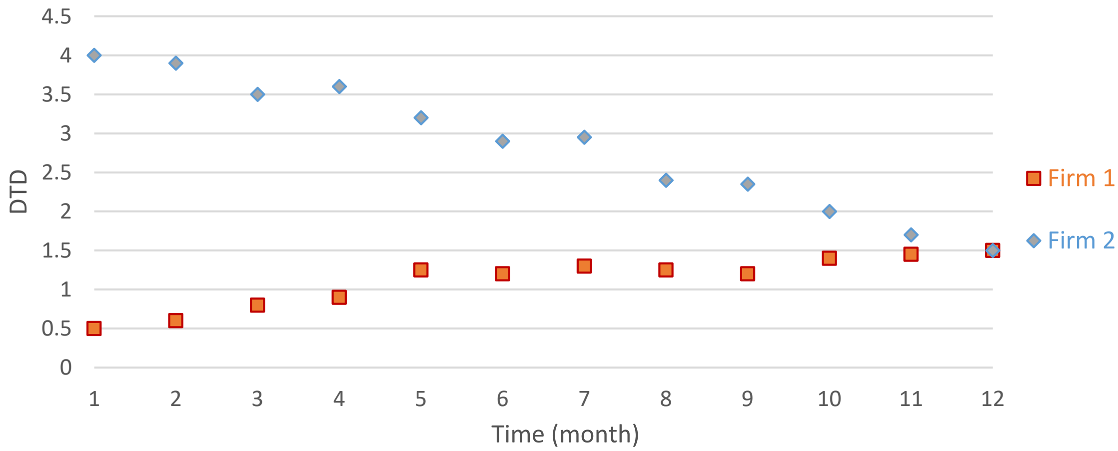


Figure 3: DTD Trend

Distance-to-Default (DTD)

KMV, now part of Moody's Analytics, first introduced the commercial usage of DTD, which serves as the foundation of its Expected Default Frequency model. Empirical studies have shown that DTD is among the best predictors of default.[3] CRI uses DTD as one of its inputs in its PD model. While conceptually resembling KMV's DTD, there are differences in the calculation of CRI's DTD to incorporate liabilities more holistically and make it better applicable to financial firms, as described later in this section.

For now, to facilitate our description, consider a simplified example of two firms in Table 4, displays financial data for financially weak and strong firms.

Table 4: A Simplified DTD Example

	Weak Firm	Strong Firm
Assets ($) (current value)	110	200
Liabilities ($) (promised amount in the future)	100	100
Volatility of Assets	20%	20%
Distance-to-Default (simplified)	10% / 20% = 0.5	100% / 20% = 5

Which firm is more likely to default? First, let us look at the health of the weak firm. It has assets of $110 and liabilities of $100. Currently, the value of the assets covers the value

[3] Studies which have used DTD as a default covariate/predictor include Crosbie and Bohn (2002), Vassalou and Xing (2004), Duffie et al (2007), Bharath and Shumway (2008), and Duan et al (2012), to name a few.

of the liabilities (after discounting) adequately. The value of assets, however, fluctuates over time due to several reasons. Among them, adverse business conditions or ineffective collections could reduce the amount of account receivables. Changing business circumstances may render some of its fixed assets devalued. Securities held by the firm, such as bonds or stocks may lose value. Over time, there is no guarantee that the value of assets will remain at or exceed $100 at the time of liabilities due so that all liabilities can be fully met. This lies behind the option-theoretical basis for DTD in the Merton (1974) model, where the liabilities, i.e. the promised payment, serves as the strike price of a call option. In the event in which the promised payment is not fully met, the call option is said to finish out of the money. The uncertainty over how much the asset will be worth is summarized in “volatility of assets”, which is 20% in this example.

DTD attempts to factor in the volatility of asset values. It measures how much headroom assets hold over liabilities per unit of volatility in asset value. For Weak Firm, assets exceed liabilities by 10 percent, and the volatility of assets is 20 percent, which yields a DTD value of 0.5 (10 percent / 20 percent). In other words, excess assets over liabilities are sufficient to buffer a 0.5 standard deviation shock to the value of its assets. Likewise, for Strong Firm, the DTD is 5 (100 percent / 20 percent) which is ten times as high as that of Weak Firm. Strong Firm’s assets can absorb a shock of up to 5 times the standard deviation of its asset value. Therefore, Strong Firm is “further” away from default than Weak Firm, and correspondingly, we expect it to have a lower PD.

The above example is extremely simplified, so several clarifications are in order. First, we are looking at the market value of assets, not the book value of assets. Book values were recorded at point of entry. They can be quite outdated and fail to reflect latest valuations. Second, the actual formula for DTD used in practice and in the CRI-PD model is more complicated, involves the use of logarithm and square root scaling to the appropriate time horizon. For completeness, we show below the DTD formula at time t adopted by the CRI system for a firm whose time-t asset value is V_t, liabilities due at time T is L, and volatility rate is σ.[4] However, our aim here is to convey the intuition rather than to cover the finer points.

[4]We have purposely left out the risk-adjusted drift term, i.e. $(\mu - \frac{\sigma^2}{2})(T-t)$, in the original DTD formula to obtain a practically more informative DTD. This is because one cannot estimate μ with a reasonable precision due to the high noise-to-signal ratio inherent in typical daily stock returns, a well-known fact in the financial time series literature. For a more complete discussion on the DTD formula, we refer readers to Duan and Wang (2012).

$$\mathrm{DTD}_t = \frac{\ln\left(\frac{V_t}{L}\right)}{\sigma\sqrt{T-t}}$$

Asset and liabilities in DTD

Let us dig one level deeper. How can we estimate the market value of assets and the promised payment, i.e., some sort of book value of liabilities for the purpose of computing DTD given the complex capital structure in reality?

To start, one must first determine what $(T-t)$ to use in practice. For no apparent reasons, the common practice has settled for one year. But obviously, liabilities for typical firms will scatter a wide maturity spectrum, and hence there is a need to apply some *ad hoc* but sensible adjustment to turn liabilities into a pseudo promised payment (referred to as default point hereafter) in 1-year time horizon. Now, since DTD looks ahead over a 1-year horizon, we can arguably count short-term debts (due within a year) directly in the default point. However, it is only reasonable to subject long-term debts (due beyond one year) to a haircut in order to conform to the 1-year horizon. The practice advanced by KMV and adopted widely in the credit literature is to haircut long-term liabilities by 50% before adding them to the default point, reflecting the fact that long-term debts are due later than one year.

The CRI model has incorporated an *additional* component into the default point. Financial firms are notorious for being difficult to model because the major portion of their liabilities cannot be classified in the short-term nor in the long-term debt category. We refer to these additional and rather large amount of liabilities as *other liabilities*. Within this category, a predominant component for banks is, of course, customer deposits which make up the bulk of any commercial bank's total liabilities. To substantiate the point, it is not uncommon for banks to be leveraged 10-20 times, with the major portion of the liabilities falling under – you guessed it – *other liabilities* as deposits. For insurance companies, other liabilities will mainly constitute of policy obligations.

Prior work in the credit literature typically pays scant attention to the role and impact of other liabilities, which are not particularly large in magnitude, vis-a-vis short-term and long-term debts, for most non-financial firms. When financial firms are conventionally excluded from default analysis, ignoring other liabilities becomes lesser of an issue.[5] In

[5] At odds with the typical finding, Campbell et al (2008) concluded that DTD does not help default prediction. According to Zou (2016), their conclusion was due to the inclusion of financial firms in the sample while leaving out other liabilities. Zou (2016) showed that using either the DTD with the KMV default

contrast to most studies, the CRI model, following the approach of Duan et al (2012), directly factors in other liabilities by adding to the default point formula a fraction of other liabilities, where the haircut rate is treated as an unknown parameter to be estimated along with other model parameters. One way to think about this issue is that customer deposits of a bank represent debt obligations, a portion of which may be withdrawn on short notice while others may stay with the bank for a long term. To subject them to the same 1-year debt horizon, haircutting is a must but its magnitude cannot be arbitrary. Setting the haircut rate as an unknown parameter and letting the data determines its suitable value seems the most logical way of proceeding. Overall, the modified default point formula given below enables CRI to expand the coverage and produce PDs for financial firms in addition to non-financial corporations.

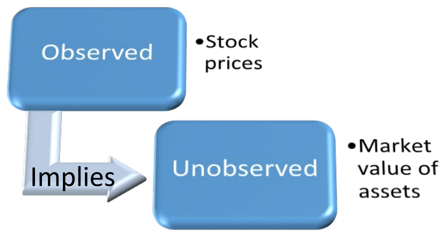
$$\text{DTD Liabilities (Default Point)} = \text{(Short-Term Liabilities)} + 0.5\ \text{(Long-Term Liabilities)} + \delta\ \text{(Other Liabilities)}$$

Moving on, our next challenge is to estimate the market value of a firm's assets. Because market values cannot be observed directly, we must rely on indirect methods to measure it. To draw an analogy, we cannot see the healthiness of a person's heart directly, not unless we surgically remove and comb through it for blockages and clots. We are not completely helpless, however, as we could instead measure his cholesterol level, check his blood pressure and obesity level, run an ECG, and interview his lifestyle habits to draw a fairly accurate diagnosis of his heart condition without looking directly at his heart (or removing it). In the same way, we can assess the market value of a firm's assets by observing its traded stock price. If the stock price rises, it should mean that the market value of assets rises. Why? Because a common stock is in fact a call option-like claim on the firm's assets. More precisely, a stock is a claim on the residual value of the firm's assets after its creditors (debt holders) have been paid off.

So, there is information pertaining to market value of assets within stock prices, and all we need is a suitable technique to extract it. For those who are familiar with options, viewing stocks as call options points to a way out. These options are written on the market value of assets with the liabilities (default point in our adopted jargon) being their strike prices. Exploiting this fact, we can infer market values of the assets from the time series of stock prices by using the option pricing formula to invert them. For finer points and references on the estimation method, we refer readers to Duan and Wang (2012). If this

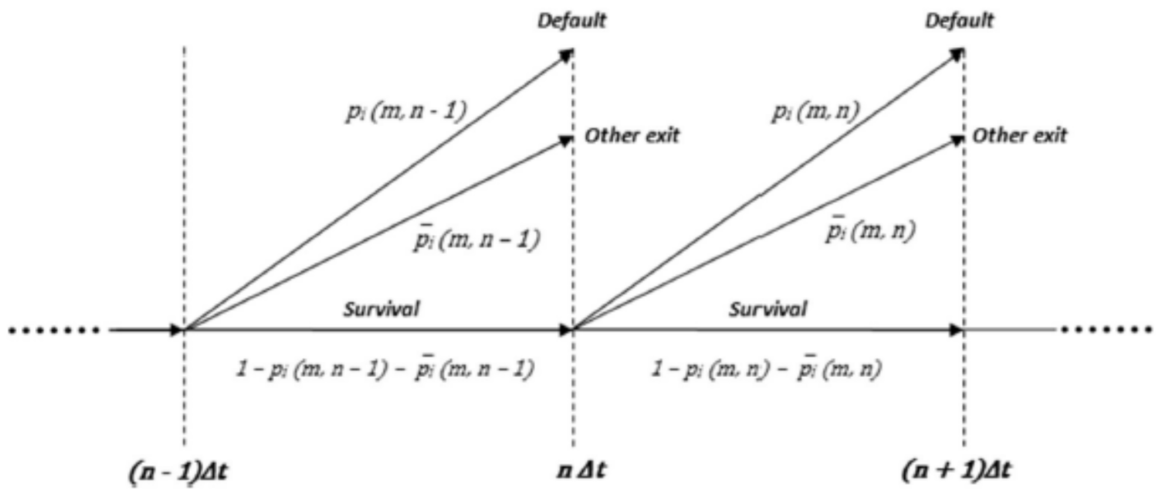
point formula on the sample exclusive of financial firms or the DTD computed with a revised default point formula that factors in other liabilities will reverse the conclusion of Campbell et al (2008).

sounds too technical, the main takeaway here is that market values of assets, while not observable, can be extracted from a time series of stock prices by reverse engineering. It is like looking for the smoke to find out where the fire is.



Modelling the Default Process

In order to utilize the data collected, we must translate the covariates/predictors into default and other-exit probabilities, keeping in mind that we observe only the actual defaults and other forms of corporate exits. Imagine for a moment that time can be split up into discrete periods as depicted in the diagram below. In each of these periods, one of three events may occur. First, the firm survives and continues as a viable entity in the next period. Second, the firm may default, in which case a "default" event is observed. Third, the firm may be delisted or merged with another firm, in which case we observe an "other-exit" event. We include this third category of events because they are a key component to measuring survival likelihood and vital to the estimation of default probabilities free of survival biases in a multi-period environment. In some economies as documented in Duan et al (2012) and the NUS-CRI Technical Report (2021) update 1, "other exits" can be up to ten times as many as exits due to default.



An attentive reader may realize that we are trying to associate the aforementioned covariates/predictors with the period by period probability of default. If the situation for the firm is unfavorable, for example, DTD is low, and liquidity and profitability are poor, it translates into a higher probability of default for the firm. However, it is important to note that firms may still survive when the situation is bad. Being a matter of chance, the firm may or may not actually default in the current period. They are simply *more likely* to default. If the bad situation persists for a period of time, it is more probable that the firm will default eventually. In this manner, the evolution of the covariates/predictors over time may be related to the actual realized defaults.

This intuition can be formalized as a Poisson process, which is a common method for modelling the occurrence of a rare event (default or other exit in this case) as time evolves. The CRI model, based on Duan et al (2012), takes in the set of covariates/predictors and combines them into time-varying forward *default intensities*, which reflect default and other-exit events being generated via a pair of Poisson processes for each firm as explained in Figure 4, and their intensities are correlated through dependency among covariates/predictors. Across firms, different pairs of intensities may also be correlated via their covariates/predictors.

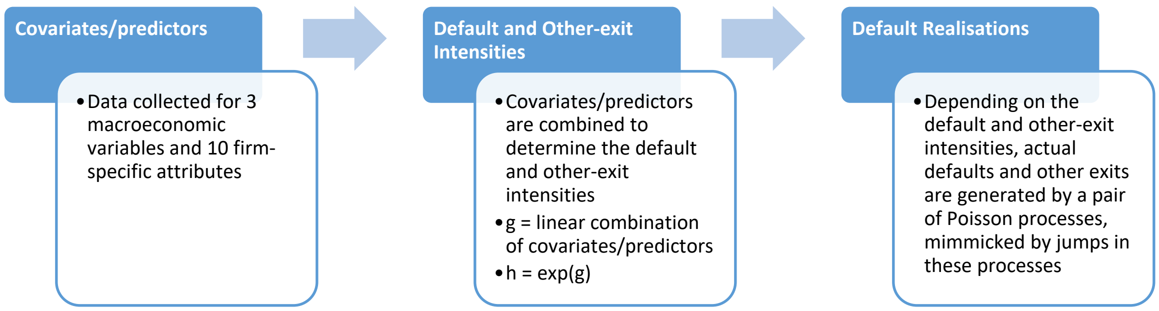


Figure 4: Modelling of Defaults and Other Exits Using a Pair of Poisson Processes

Estimation

How do we combine the covariates/predictors into the forward default/other-exit intensity? Until now, we have not specified exactly how this is done. A straightforward way to combine the information coming from each of the covariates/predictors is simply to add them up, in other words, compute:

Equation 1: Linear composite of PD input variables

$$
\begin{aligned}
\boldsymbol{g} = \quad & \boldsymbol{\beta_0} + \boldsymbol{\beta_1}\,(\text{Stock index}) + \boldsymbol{\beta_2}\,(\text{Short-term interest rate}) + \boldsymbol{\beta_3}\,(\text{Aggregate DTD}^{6}) \\
& + \beta_4\,(\text{Distance-to-default Level}) + \beta_5\,(\text{Liquidity Level}^{7}) + \beta_6\,(\text{Profitability Level}) \\
& + \beta_7\,(\text{Relative size Level}) + \beta_8\,(\text{Relative market-to-book ratio}) + \beta_9\,(\text{Idiosyncratic volatility}) \\
& + \beta_{10}\,(\text{Distance-to-default Trend}) + \beta_{11}\,(\text{Liquidity Trend}) + \beta_{12}\,(\text{Profitability Trend}) \\
& + \beta_{13}\,(\text{Relative size Trend}).
\end{aligned}
$$

Next, having a negative default intensity does not make sense, since a presently surviving firm can only default or survive in the next period; it cannot "anti-default" as there is no such interpretation. Therefore, the default intensity is set to $h = \exp(g)$, or taking an exponential of the weighted sum of individual covariates/predictors to ensure that the intensity is positive.

Estimating the model is then a matter of choosing the parameters $\beta_0, \beta_1, \beta_2, \dots, \beta_{13}$ so that the model best matches the data (observations of actual defaults/other exits). Briefly, this involves writing the likelihood of the model given the data and maximizing it

[6] β_3 is different for financial firms vs. non-financial firms, as the aggregate DTD measures are different for the two groups. See NUS Credit Research Initiative technical report version: 2021 update 1.

[7] β_5 *and* β_{11} are different for financial firms vs. non-financial firms, as the liquidity measures for the two groups are defined differently. See NUS Credit Research Initiative technical report version: 2021 update 1.

over the parameters. To grasp the intuition, consider the parameter β_6 for profitability. Since defaults tend to occur when profitability is low, we expect β_6 to be negative. A positive β_6 would mean that the default intensity had increased with profitability, which turns out disagree with what the data says. In other words, it is not very likely for β_6 to be positive. The optimization method picks β_6 and all the other β co-efficients simultaneously so that they best agree with what we see in the actual defaults/other exits. A full statistical treatment would be too involved here, but the key takeaway is that the parameters are selected to explain the default data as much as possible.

Recall that CRI produces a term structure of PDs from 1 month up to 5 years. To achieve this, we need to estimate not only a single set of parameters $\beta_0, \beta_1, \beta_2, \dots, \beta_{13}$, but an entire term structure of them up to 5 years into the future. Intuitively, it might sound infeasible or impossible to estimate the parameters for every time point into the future up to 5 years. But it is actually doable via a decomposability result established in Duan et al (2012). Naturally, we would expect adjacent horizons to share similar parameter values, and hence we can assume that the term structure of the parameters follows a particular class of curves. So using β_6 as an example again, we assume that their values across the various forward-looking horizons can be described as a curve, then instead of estimating each and every β_6, we estimate the shape of the curve which turns out to be computationally more challenging but intuitively more desirable. Apart from smoothing the parameter estimates, an added benefit is that the parameter curve allows for easy extrapolation into longer horizons for which data may not even exist. Exactly which shape this curve takes is a matter of choice. For example, we may simply decide to fit a quadratic (polynomial) curve, but this form may perform poorly. Duan et al (2012) deploys a Nelson–Siegel curve and so does the CRI-PD model. While less widely known, the Nelson-Siegel curve is adequately simple and works well. This approach is very similar to how the term structure of bond yields is modelled.

Calibration Groups

Earlier we mentioned that the CRI coverage spans over 80,000 firms. With such an extensive diversity in the sample, it is only natural to expect that the parameters $\beta_0, \beta_1, \beta_2, \dots, \beta_{13}$ differ across economies. At the same time, one must strike a balance with the need to have sufficiently many defaults in the sample to conduct model estimation/calibration. The CRI system thus deploys calibration groups by putting together firms from different economies that are expected to be more similar than not.

Currently, Canada and the US belong to the North America calibration group, and the developed economies of Asia-Pacific (Australia, Hong Kong, Japan, Singapore, South

Korea, Taiwan, and New Zealand) form another calibration group. China and India, the two major emerging economies of Asia are calibrated as two individual groups. All European countries covered by CRI are in a single calibration group. The other emerging economies of Asia Pacific, Latin America, Middle East and Africa form the "emerging markets" calibration group.

As a general rule, all economies in the same calibration group share the same coefficients for all variables except for the short-term risk-free interest rate[8]. The short-term interest rate variable is entered as the current value minus the historical month-end mean in order to reflect the contemporary change relative to the historical average. Its coefficient can vary across economies because different economies with different currencies may have different dependencies on their interest rates, the levels of which can also differ significantly across economies. The Euro zone deserves a special explanation, where all countries after joining the Euro zone begin to use German short-term interest rate.

Prediction Accuracy

CRI conducts tests to ascertain that the PDs are informative of potential defaults. A popular standard in assessing the discriminatory power of a rating system is to use Accuracy Ratio (AR). The intuition behind AR is that if firms with high PDs are indeed those defaulted ones, then AR is high. In other words, the distressed firms have been properly discriminated or distinguished from the safe firms. On the other hand, if PD levels have little to do with defaults, then AR is low. AR ranges from 0 to 1, with 0 indicating a completely uninformative rating system, and 1 a perfect system. The use of AR is widespread in, for example, documents issued by the Basel Committee on Banking Supervision and others.

Table 5 reports the ARs for various prediction horizons and economies. For example, the procedure for 1-year AR calculation is as follows: first, we calibrate the parameters using the full data set (in the tables below, this is until May 2021). Next, standing at a particular point in time, say 31 December 2000, we extract the PD forecasts 1 year ahead for all firms based on these parameters as well as the actual defaults that occurred in 2001. We subsequently pool the PD forecasts across all time points for this economy and compare them against the actual defaults to compute the 1-year AR.

[8] As exceptions, Eurozone uses German interest rate owing to their economic integration. In addition, Indonesia has its own coefficient on relative size.

Table 5: Accuracy Ratios for the CRI-PD Model

	AR			
Economy	1mth	1yr	2yr	5yr
Argentina	0.82	0.707	0.613	0.385
Australia	0.842	0.677	0.552	0.358
Brazil	0.853	0.798	0.735	0.612
Canada	0.943	0.832	0.715	0.57
China	0.71	0.681	0.644	0.59
Denmark	0.903	0.817	0.67	0.546
France	0.848	0.739	0.672	0.584
Germany	0.861	0.713	0.617	0.495
Hong Kong	0.76	0.574	0.484	0.324
India	0.738	0.707	0.683	0.607
Indonesia	0.709	0.679	0.647	0.527
Italy	0.879	0.811	0.675	0.546
Japan	0.906	0.858	0.814	0.691
Malaysia	0.789	0.754	0.695	0.54
Mexico	0.795	0.762	0.711	0.605
Netherlands	0.882	0.843	0.717	0.493
Norway	0.957	0.844	0.722	0.517
Philippines	0.657	0.633	0.625	0.627
Poland	0.865	0.739	0.639	0.414
Russian Federation	0.637	0.466	0.256	0.117
Singapore	0.663	0.704	0.582	0.305
South Africa	0.918	0.841	0.722	0.455
South Korea	0.89	0.762	0.709	0.637
Spain	0.829	0.616	0.485	0.412
Sweden	0.925	0.848	0.772	0.565
Taiwan	0.929	0.822	0.739	0.624
Thailand	0.848	0.811	0.765	0.666
UK	0.872	0.744	0.617	0.422
US	0.941	0.86	0.767	0.611
Developed Asia-Pacific	0.857	0.76	0.685	0.551
Emerging MKT	0.804	0.757	0.704	0.583
Europe	0.87	0.752	0.647	0.486
North America	0.941	0.857	0.761	0.606

*The table is taken from CRI technical report 2021 update 1.

The CRI model can achieve strong AR results mostly greater than 80% at the one-month horizon, with stronger results for developed economies. At 1-year, ARs are mostly healthy and above 70%. There is a drop in AR at the 2-year and 5-year horizons, but this is to be expected as we move further on in the term structure.

The ARs in some emerging economies such as India, Indonesia, and the Philippines are noticeably weaker than the results in developed economies. This can be due to several issues. The quality of data may be worse in emerging markets, in terms of availability and data errors. This may also be due to lower reporting and auditing standards. Furthermore, the variables selected are likely to play a more important role in emerging markets. The variables in the CRI implementation thus far were mainly selected based on the predictive power in the original US implementation in Duan et al (2012). Had the demanding variable selections been conducted for different calibration groups, we could expect improvements to their predictive accuracies, especially for emerging economies. Finally, there could be structural differences in how defaults and bankruptcies occur in emerging economies. If the judicial system is weak and there are less repercussions for default, firms may be more prone to default.

Previously, China's AR for 1-year PD was 57%, but has improved to 67% with the introduction of structural break estimation. The structural break occurs in December 2004, and we allow the coefficients for DTD level and the intercept to be different after this break. However, we incorporate a modification to the standard structural break estimation. Instead of a sudden change in these coefficients, we let the parameters change smoothly into the new parameters over time[9], reflecting gradual changes rather than being brought about by a sudden shift in economic structure. Going beyond mere risk ranking firms, this structural break treatment delivers a meaningful improvement in predicting realized default rates experienced by the sample of Chinese firms. Currently, we model the structural break by a step function allowing for different rates of transition to and away from the break point. The treatment is the same for the intercept term and the coefficient for the DTD level, but the transition rates are different.

Furthermore, a state-owned enterprise (SOE) dummy variable is introduced to the model on the sample of Chinese firms, starting from Apr 2021, to reflect the fact that Chinese SOEs are generally perceived by the market to be "safer" as compared to their non-SOE counterparts. This additional variable has delivered improvement in the predicting realized numbers of defaults for both SOEs and non-SOEs.

The North America calibration group (the US and Canada) has incorporated two specific changes. First, a dummy variable on the intercept for financial firms is included to account for differences that have not been duly reflected through over covariates. Second, a structural break, which is treated as an impulse response, is applied to this financial-sector intercept dummy to address the change in September 2008 after Lehman Brothers' default. After incorporating the two specific treatments, the AR for the 1-year CRI-PD (calibrated in April 2018) increases from 84% to 86%. Although the increase in the AR is

[9] This is implemented using a logistic function which starts with the old set of parameters and tends to the new set of parameters as time goes on. See NUS Credit Research Initiative technical report version: 2021 update 1.

not pronounced, the revised model has clearly delivered a meaningful improvement in predicting realized default rates for the North America calibration group.

ARs are good for determining if PDs rank the firms correctly according to their relative default risks. For example, a riskier firm should have a higher PD. There is a slight catch, however. Theoretically speaking, even if we multiplied all our PDs by a factor of say 2 or 3, the accuracy ratio would remain unchanged. Therefore, we also want to find out if the actual PD level, say 5%, actually reflects a 5% default risk. In other words, we want to assess the goodness of fit of the PDs. This is done by comparing the predicted and actual number of defaults at the aggregate level over time. Standing at a particular time point, we ask how many firms are expected to default in the next year, which we can compute using the forward PDs that have already been estimated at that point. We then compare this number with the actual number of firms defaulted over the next year. Figure 5 shows this for the North America group and Figure 6 for China[10]. The predicted levels track the actual levels quite nicely, bearing in mind that the numbers are predicted in advance of the actual defaults. Most of the other economies exhibit a similar pattern.

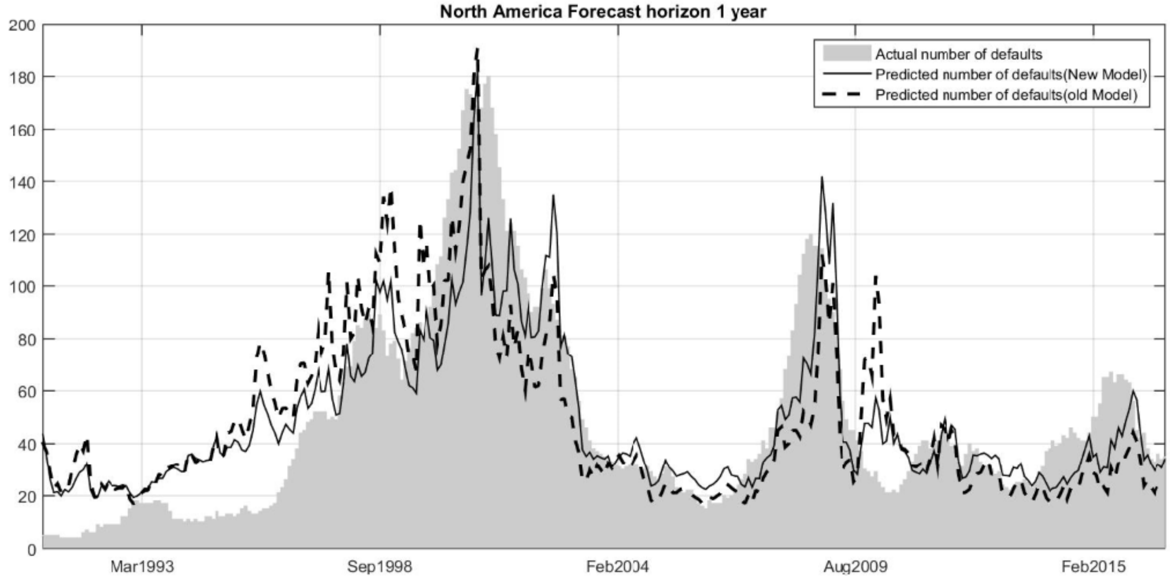


Figure 5: Predicted and Actual Defaults for United States

[10] Figure 5 and Figure 6 are from CRI staff calculations. The structural break treatment for Chinese firms was implemented in two phases. The marked improvement in AR from 57% to 67% described earlier reflects the change in the first phase. The old vs new models in Figure 6 are the change implemented in the second phase.

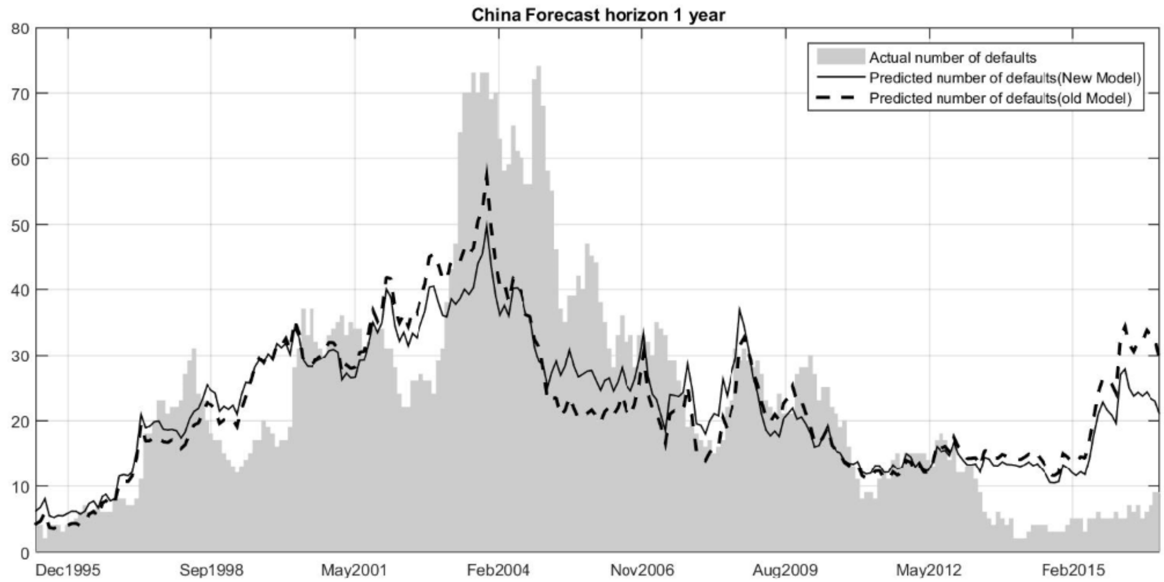


Figure 6: Predicted and Actual Defaults for China

Alternatives for CRI-PD

We now briefly mention a few alternatives which may be used to assess the credit quality of a company.

Letter credit ratings issued by credit rating agencies (CRAs) such as Standard & Poor's, Moody's Investor Services, and Fitch Ratings are possibly among the most widely used. For example, the AAA/Aaa grade is the highest possible rating and represents companies with extremely low credit risks. The opposite is true for companies rated CCC. While popular and easy to understand, criticisms of such letter credit ratings have arisen particularly after the financial crisis of 2007-2009. One topic is the lack of responsiveness to changes to the company’s circumstances. Indeed, Lehman Brothers was rated as an investment grade company with at least an A rating by the big three rating agencies up until mid-September right before its bankruptcy. Another criticism is that CRAs have a profit incentive in the companies which they rate, leading to potential conflicts of interest, as noted in The Financial Crisis Inquiry Report of 2011 prepared by the Financial Crisis Inquiry Commission.

In comparison, CRI-PDs are more responsive, being updated daily and recalibrated monthly. The model incorporates market data such as the company's DTD, the stock market performance, and prevailing interest rate, among other information. CRI-PDs are provided as a public good and distributed as a free service with the intention of promoting the use of a scientifically rigorous credit risk measure and facilitating research and development in credit risk analysis.

More closely related to CRI-PDs are Moody's Expected Default Frequency (EDF) database and Kamakura's PDs, but neither of them intends to measure the probability that a firm will default over a specified period. According to available information, Moody's EDFs primarily incorporate DTD but without further technical details on its model, whereas the exact nature of Kamakura's method is unclear to us. Both Moody's EDFs and Kamakura's PDs are paid services. CRI-PDs, on the other hand, incorporate not only DTD, but also a range of common risk factors and firm-specific attributes as explained earlier, and the methodology is fully disclosed to the public.

Summary of CRI-PD

PD tells us how likely it is for a firm to default over a given period in the future and conveys a more granular sense of the credit quality of a company. We can use PDs to study a range of topics, including questions pertaining to financial stability. CRI-PDs have the advantages of global coverage, a term structure of up to 5 years, and the methodology is fully disclosed, and PDs are freely accessible via the CRI website. Being a non-profit undertaking, CRI does not bear an inherent conflict of interest with profit incentives.

The CRI-PD model uses the Poisson process to model corporate survivals and is calibrated against actual defaults and other exits. The default and other-exit intensities for the Poisson processes are linked to 3 common risk factors and 10 firm-specific attributes, namely, stock index, short-term interest rate, Aggregate DTD, relative market-to-book ratio, idiosyncratic volatility, DTD, liquidity, profitability, and relative size, with the latter 4 having both level and trend variables. DTD is particularly helpful for predicting defaults, and our modification of incorporating customer deposits and insurance policy obligations (other liabilities) makes it work better on financial firms and in those applications involving credit portfolios formed of both financial and non-financial firms.

BuDA

Equipped with a basic but workable understanding of CRI-PD model, we will in this section explore the conceptual underpinnings of the bottom-up default analysis (BuDA). Implementation details and operation of the toolkit will be left for the additional document, BuDA operational guide[11]

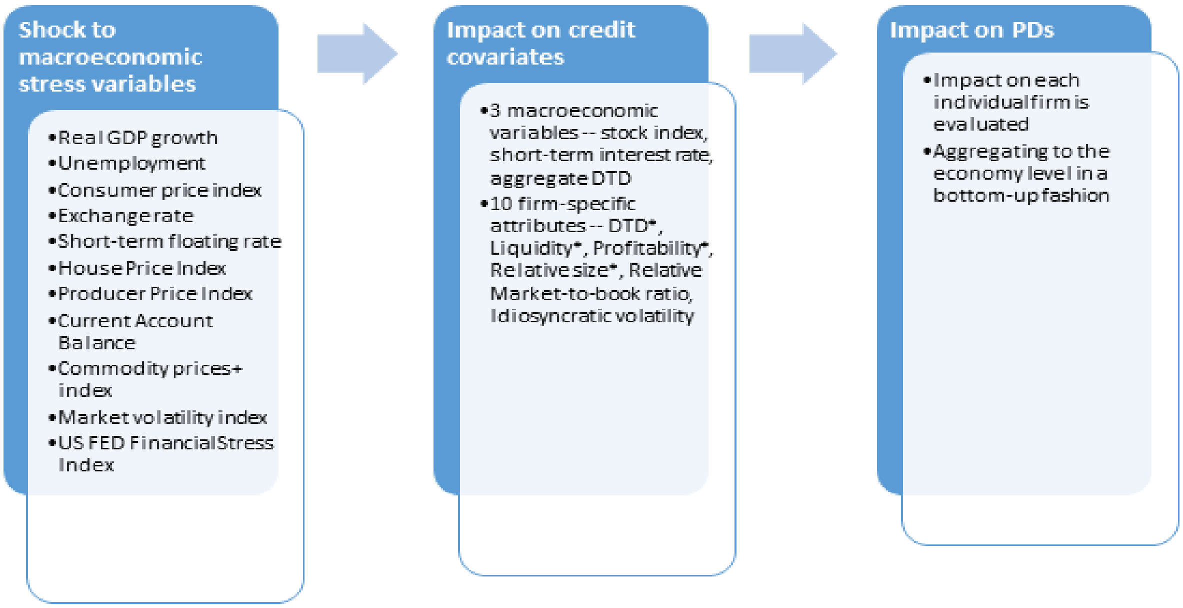
What is BuDA?

BuDA is an approach developed by Duan et al (2014) to conduct credit stress testing and scenario analysis that relates shocks to macroeconomic stress variables, such as GDP, unemployment, inflation and others, to the credit quality of the economy and/or industry. For example, an economist might be interested in analyzing the health of the economy when a prescribed severe recession, i.e. consecutive falls in GDP, occurs. BuDA translates the specified fall in GDP into an impact on the default covariates/predictors discussed in the previous section, which in turn determines the impact to the PDs of the targeted firms, see Figure 7. In other words, starting with the macroeconomic dimensions which an economist might be concerned with, BuDA interprets these shocks in terms of their corresponding effect on the credit quality of the target portfolio. BuDA can be useful to regulatory authorities, central banks, and commercial/investment banks for increased awareness and understanding of risks.[12]

As one might expect, the macroeconomic variables of interest to an economist may not coincide neatly with the default covariates/predictors – 3 common risk factors and 10 firm-specific attributes which are used in the CRI-PD model, covered in the previous section. The idea is therefore to map the impact of the macroeconomic shocks to these default covariates/predictors, and in turn to measure the follow-on impact on PDs. Let's use some math, through which we can convey the intuition more concisely.

[11] The Credit Research Initiative team (2021), Bottom-up Default Analysis (BuDA v3.3.1), The user manual of BuDA toolkit, Accessible via https://client.nuscri.org/static/asset/BuDAOperations.pdf.

[12] The BuDA toolkit allows for the deployment of user-supplied stress variables other than the 12 listed in Figure 7. It also aggregates any user-targeted portfolio instead of the pre-specified economies.



(*) indicates the variables with both level and trend

Figure 7: Overview of BuDA Approach

Equation 2: Stress Testing Regressions

Change in common risk factors | Stress variables | Lagged common risk factors

$$\Delta X_{m,t} = \beta^X_{m,0} + \sum_{k=1}^{n} \beta^X_{m,k} Z_{k,t} + \gamma^X_{m,1} X_{m,t-1} + \gamma^X_{m,2} X_{m,t-2} + \varepsilon^X_{m,t}$$

$$\Delta \bar{Y}_{i,j,t} = \beta^Y_{i,j,0} + \sum_{k=1}^{n} \beta^Y_{i,j,k} Z_{k,t} + \gamma^Y_{i,j,1} \bar{Y}_{i,j,t-1} + \gamma^Y_{i,j,2} \bar{Y}_{i,j,t-2} + \varepsilon^Y_{i,j,t}$$

Random error terms

Change in firm-specific attributes | Stress variables | Lagged firm-specific

As illustrated in Equation 2, changes to the default covariates/predictors are driven by the macroeconomic stress variables. Since some of the covariates/predictors exhibit persistence, we include lag terms to take care of autocorrelation. For those who are familiar with autoregressive systems, the model implemented above is an AR(2) model, i.e. 2 lags. This linkage to macroeconomic stress variables of interest affords flexibility of choice and effectively decouples the variables for stress consideration from the covariates/predictors that work for default prediction.

Now pay attention to the subscript for the macroeconomic stress variables on the right side of the equation, then, look at the subscript for the change in default covariates/predictors on the left side. Notice that both have the subscript *"t"*. This means that information pertaining to the macroeconomic stress variables enters the default covariates over the same period with no lag. In other words, the default covariates/predictors will respond at the same time, and we are interested in macroeconomic stress variables' *contemporaneous effect* on the default covariates/predictors.

We take this opportunity to also point out the random error terms at the end of each equation. As much as we attempt to predict the impact which macroeconomic stress variables may have on the default covariates/predictors, there is an element of uncertainty, and it is only wise to recognize it. Consequently, the model is not limited to predicting a single deterministic outcome under the prescribed macro scenario but can predict a range of possible outcomes. Later, we will use the above equation to simulate the system over the intended horizon of interest by drawing random numbers for the error terms.

We need to clarify further with regards to the second regression pertaining to changes in firm-specific attributes. Recall that we have a dataset for over 85,000 firms, and a good proportion of these do not have a long history of data for stable parameter estimation. Also, estimating the regression for each of them might entail additional sampling errors which would be impounded into the final results. Even if we are only focusing on specific regions, the US has over 16,000 firms, while the Eurozone 12 has over 7,000 firms and ASEAN 5 spills slightly over 4,800. To accommodate a large number of firms and for feasibility sake, we need some simplification and conduct the stress testing regressions only for the *mean* of each firm-specific attribute $(\bar{Y}_{i,j,t})$, where the *mean*[13] is taken across firms in each of 12 industries where there are 9 non-financial sectors and the financial industry is further divided into three sub-sectors.

To deal with individuality, the *mean* is subtracted from the individual firm's attribute values to obtain many residual series $(Y_{i,j,t} - \bar{Y}_{i,j,t})$. This determines how far off an individual firm is from its industry benchmark, i.e., a *relative position*. We then model each relative position series as an AR(3) process for the last 24 months prior to the start of a testing scenario. However, missing values for some firm-specific variables will inevitably occur, causing the effective sample size to be unreasonably small. We thus adjust the autoregressive order to reflect the sample size. Specifically, the p in the AR(p) model takes a value of 3 if the sample size larger than 17, 2 if it is in the range of 12 to 17, 1 if it is in the range of 6 to 11, and 0 if it is below 6.

The stress-testing regression model for the mean and the residual's AR(p) dynamic are jointly employed to obtain future simulated values for each firm-specific attribute under the specified macroeconomic scenario. Apart from the statistical consideration explained above, the computational benefit arises from only conducting stress testing regressions for 12 series of cross-sectional mean values for each of the firm-specific attributes.

Stress Variables

Table 6 below lists and briefly describes each of the standard variables implemented within BuDA. Users may define their own macroeconomic stress variables, but for now, let us focus on the standard ones (i.e., the default choice). BuDA provides the historical data for eight country-specific macroeconomic variables together with three country-specific common factors. In additional, BuDA also provides three groups of other potential stress variables of interest plus credit cycle indices generated from CRI-PDs.

[13] More precisely, we use the 20% trimmed mean, in other words, the top and bottom 20% of observations are dropped before taking the mean. Moreover, when there are insufficient data (less than 5 firms or less than 3 years of data) in the specific economy-sector, the aggregation group-sector mean is used as a substitution. The aggregation groups are: ASEAN, Non-ASEAN, North America, Eurozone, Non-Eurozone, Sub-Saharan Africa, Latin America, Caribbean, Non-MENA, MENA. This is because, for example, Qatar banks are more similar in nature to MENA banks rather than Qatar industrial firms as they are all regional banks.

Table 6: Selection of Stress Variables

Type	Variables	Brief Description
Country-specific macroeconomic variables	GDP	Real Gross Domestic Product growth rate
	UNEMP	Difference of Unemployment Rate
	CPI	Percentage change of Consumer Price Index
	NEER	Percentage change of Nominal Effective Exchange Rate
	INT	Difference of 3-month Interbank Rate
	HPI	House Price Index growth rate
	PPI	Percentage change of Producer Price Index
	CAB	Difference of Current Account Balance
Country specific common risk factors (CRI-PD predictors)	Stock Return	Monthly stock return
	Interest Rate	3-month Interbank Rate (level)
	Aggregate DTD	Aggregate distance-to-default for financial and/or non- financial industry
Other key stress variables	Commodity Prices	Percentage change of Standard and Poor's Goldman Sachs Commodity Index and over 20 individual commodities
	VIX	Percentage change of the Chicago Board Options Exchange Volatility Index
	FFI	St. Louis Federal Reserve Financial Stress Index (level)
Credit Cycle Indices	CCI	Credit Cycle Indices are generated by aggregating CRI-PDs. User can select the data from country to industry levels.

Real GDP reflects the state of an economy with its growth rate serving as a proxy for the growth in incomes and earnings of firms. A higher growth would generally lead to higher corporate earnings and lower default risk.

Unemployment rate affects the consumption and household spending. An increase in the unemployment rate would generally result in lower revenues for firms, particularly those consumer-oriented businesses (e.g., restaurants, retailers, etc.), and increase their default likelihoods.

Consumer Price Index provides a measure of inflation and controlling inflation rates is one of the primary objectives of monetary policy. High inflation is usually considered a signal of macroeconomic mismanagement and a source of uncertainty. Higher inflation leads to increased costs and tends to impair credit quality. However, higher inflation may also reduce debt burden in real terms, and thereby improve creditworthiness. Producer Price Index is also included to capture the cost of production for domestic manufacturing industries. Higher producer prices signal higher cost of production, which may be harmful to the general credit quality of companies.

Exchange rate affects the bottom-line of a wide range of firms directly or indirectly through trade and investment flows; for example, a stronger domestic currency will typically benefit importers and likely hurt exporters. Firms in a small open economy will be particularly sensitive to exchange rate movements. We use the BIS nominal effective exchange rate indices, which are calculated as geometric weighted averages of bilateral exchange rates. In tandem, the current account balance of economies is also provided to capture the level effect of the surplus or deficit present in the net trade position. This metric helps to measure the feasibility of further policy intervention, due to incumbent public debt levels, taken by national governments.

Short-term floating rate is a common benchmark that banks use in determining lending rates for variable-rate loans. It is also the rate that is applicable to short-term corporate funding via debt/commercial paper markets. A higher borrowing rate/cost of funding is expected to increase default risk of firms owing to higher interest expenses.

House Price Index is also provided to measure inflation specifically experienced in land/property values. They may be important indicators for users who want to measure systemic risk (like the one in the global financial crisis of 2007-2009) present in financial institutions, and more broadly, in the economy. Additionally, the Financial Stress Index produced by the US Federal Reserve Bank St. Louis helps measure systemic risk present in the US financial system, which may in turn be indicative of the risk present in the global financial network.

The S&P GSCI Commodity Index captures an important production factor cost. A higher commodity price typically benefits commodity producers but causes deteriorated creditworthiness for other companies that use commodities as inputs. Individual commodity prices are also included for users to test the impact on PDs driven by specific commodities; for example, the damages to the creditworthiness of US firms exacted by an oil price spike.

The VIX index offered by the Chicago Board Options Exchange reflects the volatility of the S&P 500 index portfolio and is a commonly used proxy for gauging the asset risk level in

US stock markets, but can also be indicative of how volatile stock markets in other economies, given the pervasive global linkage of financial markets and the US dominance.

A few words need to be said about mixed-frequency data. GDP and unemployment data are typically available quarterly, whereas the other variables are available monthly or more frequently. Had we simply treated the data as a monthly panel, GDP and unemployment rate would exhibit spikes every 3 months and flat at other times. This is of course not desirable. To address this issue, we incorporate two features into the stress testing regressions. Firstly, instead of having jumps every 3 months, the quarterly variable are linearly interpolated so that their monthly changes are gradual rather than sudden. Secondly, we re-express the stress testing regressions in Equation 2 so that the variables become overlapping 12-month smoothed aggregates instead of 1-month observations. Those who are interested in the technical details can refer to Duan et al (2014). In a nutshell, smoothing is accomplished by iteratively substituting the autoregressive formula into itself. The regression of default covariates/predictors against stress variables stated as 12-month smoothed aggregates makes the estimation much less sensitive to how the quarterly data are converted into monthly data.

Top-down vs Bottom-up

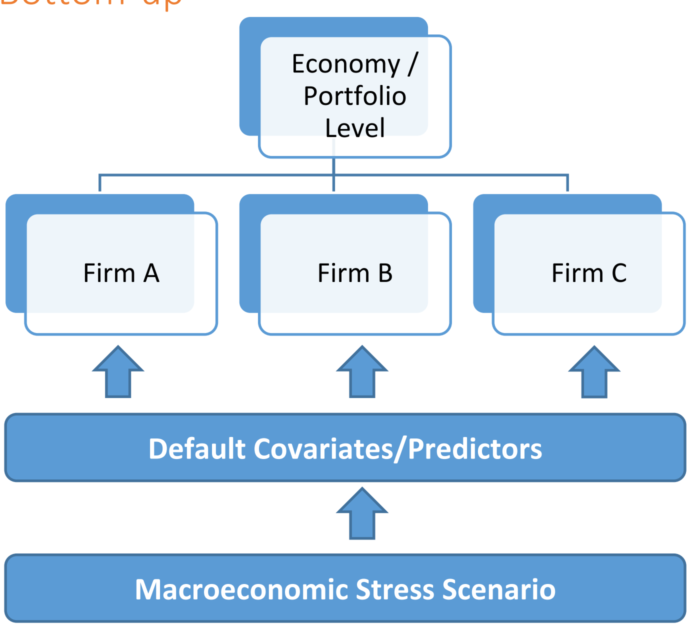


Figure 8: Bottom-up Aggregation

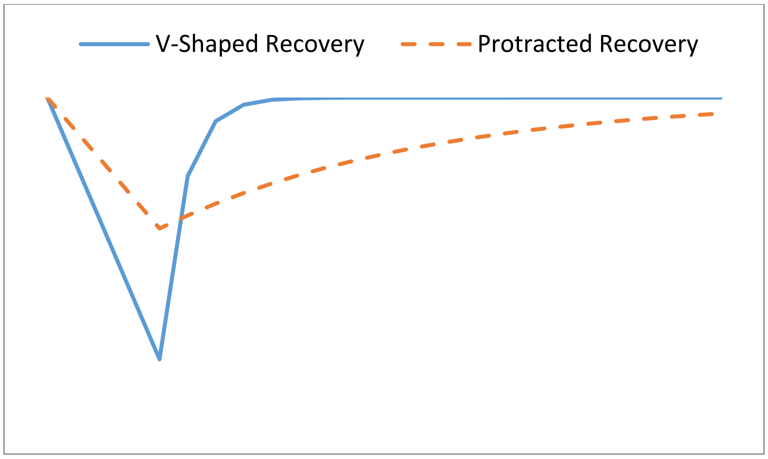
As the name suggests and the diagram in Figure 8 depicts, “bottom-up” refers to an approach where the impact on each individual firm in the pool of firms can be evaluated and aggregated up into the portfolio or economy level. The methodology computes PD impacts at the most granular firm level and builds it up to the overall portfolio level by aggregation or specifically stated, taking a portfolio-level statistic such as the average, median or some percentile.

Each portfolio aggregate PD is computed at the conditionally simulated PD and POE input variables under a prescribed stress scenario. Therefore, BuDA will approximate its theoretical value by averaging the simulated portfolio aggregate PDs over, say, 1000 simulation runs, to arrive at the final stressed portfolio-level PD. In short, BuDA outputs an expected value of the target portfolio’s average, median, or some-percentile PD.

“Top-down” scenario analysis, on the other hand, involves first consolidating the data, and then performing the scenario analysis directly on the consolidated measures. In a typical setup, a regulator or financial stability governing authority may not be able to work with granular individual firm data, and hence limits itself to using consolidated data. The BuDA approach which leverages on the vast CRI-PD dataset provides a practical bottom-up solution.

Stress Testing Scenarios

BuDA allows a range of macroeconomic stress scenarios to be specified without imposing too much limitation. One important point to note is that the stress scenario may be multi-period, occurring over several months or years instead of a single time period – in fact, the worst situations are likely an accumulation of undesirable events instead of a drastic single period event (although that can also be accommodated).



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National University of Singapore

Figure 9: Examples of GDP Growth Rates under V-Shaped Recovery and Protracted Recovery

Next, stress scenarios need not be stylized as a unidirectional shock. For the example in Figure 9, a “V-shaped” recession and recovery used by the International Monetary Fund may be applied. In this case, both the fall of GDP into recession and subsequent recovery are incorporated. Alternatively, a protracted recovery situation may also be analyzed. As can be seen later, the flexibility allows even the actual evolution of historical events to form the stress scenarios for backtesting.

Stress Variables Recommender

The BuDA toolkit provides a recommender function helping users select a proper set of stress variables that are most apt for the target portfolio. The inbuilt function can select, say, 5 out of 1000 candidate variables that best fit the movements of those PD input variables pertinent to the target portfolio. The algorithm utilizes the cutting-edge zero-norm variable selection technique of Duan (2019). In this section, we discuss the principles behind and the use of the recommender.

The first step mainly involves the preparation of data whereas the second step interacts with a user’s inputs.

For the data preparation, the BuDA toolkit will first quantify the contribution of each PD input variable (see Table 3) to the PD. Intuitively, some PD input variables play a more significant role in determining the magnitude of PD; for example, DTD has a higher explanatory power than does profitability. The inputs to the CRI-PD model can be consolidated into a linear composite of all input variables as in Equation 1. It is this linear composite on which we should focus.

The calibrated PD model appropriate for firm *j* in the target portfolio with *J* firms may come from multiple economies/sectors. Furthermore, the PD covariates/predictors used in the stress-testing regressions can be either economy/sector-specific common risk factors or individual firm attributes in the form of an industry average. We thus need an economy/sector identifier *i(j)* to reflect the fact that this identifier is actually linked to firm *j*. To measure the importance of the k^{th} PD input variable, denoted by X_k, out of a total of *K* such variables for the target portfolio[14], we assign firm *j* a weight

[14] The PD input variables considered here exclude those trend variables because they will be derived from the original variables under the testing scenario.

$$W_k^{[i(j),j]} = \left(\beta_k^{i(j)}\right)^2 Var(X_k)$$

The above weight accounts for both the magnitude of the coefficient of each PD input variable in Equation 1 and the variation of that economy-wide or sectoral average variable.

The overall impact of X_k on the target portfolio is measured by an average, $\overline{W}_k$, which is be taken over all *j*'s, i.e.,$\overline{W}_k = \frac{1}{J}\sum_{j=1}^{J} W_k^{[i(j),j]}$. The weight on each PD input variable X_k is then normalized to become

$$\omega_k^{PD} = \frac{\overline{W}_k}{\sum_{k=1}^{K} \overline{W}_k}$$

Let $\boldsymbol{\omega^{PD}}$ denote the weighting vector for the PD equation. Likewise, we can obtain a weighting vector $\boldsymbol{\omega^{POE}}$ for the POE equation. The final weighting vector deployed is a simple average of $\boldsymbol{\omega^{PD}}$ and $\boldsymbol{\omega^{POE}}$. Doing so recognizes the fact that the impact on the default likelihood over multiple monthly periods, say, one year, hinges on a complex interaction of 12 monthly forward PDs and POEs.

Our goal is to select a best common subset of stress variables that explains most of the variations in the PD and POE input variables. The selection task can be viewed as a weighted multivariate multiple regression where the multiple dependent variables (the PD and POE input variables) are regressed on the common subset of stress variables. Our variable selection aims to maximize the weighted R^2 that is computed by applying the above-described weighting vector on the R^2's of individual equations.

The selection task can only be accomplished if the target number of selected stress variables is set and a computing algorithm is available. The analytical challenge of choosing, say, 5 out of 1000 potential stress variables can be daunting. The task is commonly known as the zero-norm variable selection, which is in essence an NP-hard combinatory optimization task. With the advent of sequential Monte Carlo combinatory optimization[15], an optimal solution in the form of a Monte Carlo estimate for our selection problem can be typically obtained in under ten minutes using a modern desktop computer.

[15] For the technical details of the sequential Monte Carlo combinatory optimization algorithm, please refer to Duan (2019).

Users will be asked to define a pool of potential stress variables and to set a target number of selected variables that fits the context of his/her analysis. In addition, users can also specify some variable(s) as the 'must-include' stress variable(s) to reflect a task's special consideration. The algorithm will find other stress variables to best complement the 'must-include' stress variable(s) to form the final set of recommended variables as desired.

The BuDA toolkit will then perform stress testing per our earlier discussion by deploying the set of recommended stress variables.

III. BACKTESTING PERFORMANCE

How well does the BuDA methodology work for scenario analysis? Backtesting can help assess its suitability. In a backtesting exercise, one simply goes back in time to estimate the stressed PD using the actual realization of the stress variables over the period. Doing so allows a user to compare the stressed PDs with the actual PDs over the period.

Does the BuDA methodology perform well for different regions or industries? To illustrate this, we provide two examples with the data from different regions and industries.

First, we consider the energy industry of US, where the stress variables are US-GDP, VIX and the percentage change of the BFO Crude Oil price (OIL). Assume that we were back in time to May 2016 and interested in how the model performs over next 5 years. Does it predict the subsequent PD profile well? Since we already know what had happened to the stress variables, the predicted PDs over the same period could be calculated by the BuDA model.

To perform the test, the stress testing regressions are estimated using the training data between Jan 1990 and May 2016. Then, the testing scenario is based on the time series of US-GDP, VIX and OIL from May 2016 to Mar 2021 as the BuDA inputs. The model predicts the PD profile for the future dates. If the predicted PDs under the actual scenario are good, they should match well with the actual movements of the CRI-PD profile.

The BuDA model simulates all necessary input variables over the period from May 2016 to Mar 2021, conditional on the testing scenario specified under the actual realizations of those stress variables to obtain a possible future PD profile as at 31 May 2016. This simulation is repeated 1,000 times to obtain 1,000 possible realizations of the PD profile. The average PD profile and the actual PD profile are then compared.

The results in Figure 10 show that the stressed PD behaves similar in pattern to the actual PD. The stressed PD (red dash line) has an upward trend before hits the peak in Mar 2020 even though its magnitude is not as pronounced as that of the actual PD. The pattern reflects relatively well the impact of the exogenous shock due to the COVID-19 pandemic. The lower stressed PD vis-à-vis the actual PD may simply reflect the extraordinary fiscal and monetary measures taken by the US authorities to prop up economic activities while the markets still render a rather pessimistic assessment of the equities of US firms.

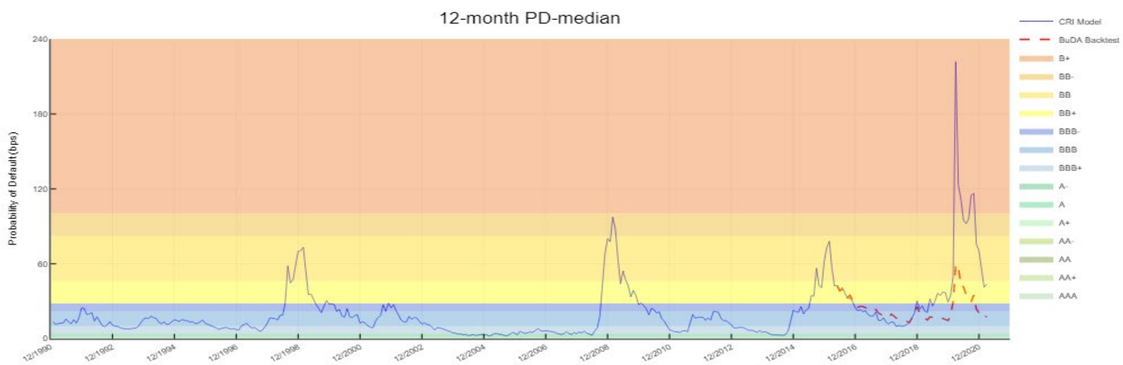

Figure 10: Backtesting Performance for the Energy Industry in US

In the second example with the results presented in Figure 11, we consider the backtesting performance for the entire UK market. However, rather than using the historical data up to the stress testing time point, the stress testing regressions for the UK sample are estimated using the whole sample period (i.e. from April 1990 to December 2020) on three stress variables (GDP, CPI and Stock Index return). Again, the stress testing scenario is defined by the actual time series of the three stress variables. The backtesting period runs between December 2000 and December 2020. The result shows that the stressed PD (red dash line) can mostly capture the trend/movement of the actual PD.

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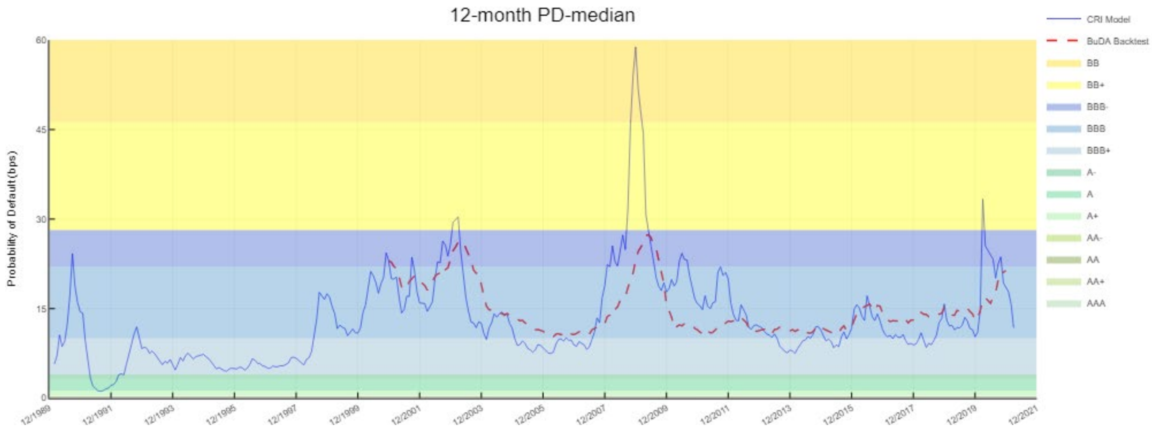

Figure 11: Backtesting Performance for the UK Economy

The final example shows how the stress variables recommender can help with the task. Figure 12 displays the backtesting results using the algorithm-recommended 5 stress variables that are selected with the inbuilt BuDA function from a list of close to 132 candidate variables for the target portfolio comprising all exchange-listed financial firms in the ASEAN-5 economies.

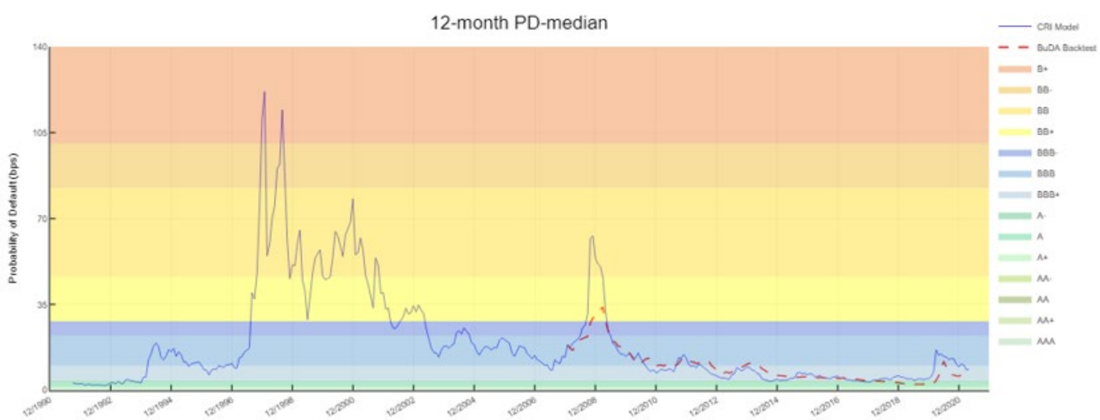

Figure 12: Backtesting result of ASEAN-5 financial industry

The results reveal an excellent performance which is largely due to identifying the 5 stress variables that are most capable of describing the movements of the PD and POE input variables in totality for the ASEAN-5 financial sector.

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IV. SUMMARY

The BuDA toolkit has been developed for the purpose of credit stress testing and macroeconomic scenario analysis, which is to investigate how individual firm PDs change under prescribed scenarios of interest. This can be useful for regulatory authorities, central banks, and commercial /investment banks for increased awareness and understanding of risks.

To estimate PDs under stress, BuDA provides a unique framework to assess the impact of macroeconomic stress variables, selected by users or recommended by the BuDA toolkit, on individual firms through the common risk factors and firm-specific attributes. With the functional relationship in place, one can simulate values for all covariates (common and firm-specific) under the prescribed scenarios and apply them to estimate the stressed PDs using the CRI-PD model. The simulated stressed credit risk profile for the portfolio of interest can then be constructed by a bottom-up aggregation.

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ABOUT THE CREDIT RESEARCH INITIATIVE

The Credit Research Initiative (CRI) was launched by Professor Jin-Chuan Duan in July 2009 at the Risk Management Institute of the National University of Singapore. Aiming at "Transforming Big Data into Smart Data", CRI covers over 80,000 public firms and produces daily updated Probabilities of Default (1-month to 5-year horizon), Actuarial Spreads (1-year to 5-year contract) and Probability of Default implied Ratings on over 40,000 currently active, exchange-listed firms in 133 economies. CRI also distributes historical time series of over 40,000 inactive firms due to bankruptcy, corporate consolidation or delisting for other reasons. In addition, CRI produces and maintains Corporate Vulnerability Indices (CVI), which can be viewed as stress indicators, measuring credit risk in economies, regions, and special portfolios.

As a further step, CRI converts smart data to actionable data to offer bespoke solutions to meet demands of its users. A concrete example is our development of the BuDA (Bottom-up Default Analysis) toolkit in collaboration with the International Monetary Fund (IMF). BuDA is an automated analytic tool based on the CRI-PD system, enabling IMF economists to conduct scenario analyses on the macroeconomic and financial linkage.

CRI publishes Weekly Credit Brief and Semi-Annual Credit Summary, highlighting key credit-related events, offering insights based on the CRI-PDs of the entities involved, and providing useful statistics on credit risk of economies and/or sectors.

NUS Credit Research Initiative
Address: 21 Heng Mui Keng Terrace, I^3 Building, Level 4, Singapore 119613
Tel: (65) 6516 3380 Fax: (65) 6874 5430
Website: http://nuscri.org/

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National University of Singapore