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Default Correlations (DC)

White Paper

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ABSTRACT

In April 2022, the NUS Credit Research Initiative (CRI) releases default-number distributions for different credit portfolios and horizons by explicitly factoring in default correlations (DCs). The DCs between corporate entities in a credit portfolio of interest are essential to producing default-number (or default-rate) distributions that go beyond marginal PDs and POEs. Default-number distributions with DCs are available for each month starting from April 2022 and updated weekly on the NUS-CRI website (https://www.nuscri.org). Periodic updates to the historical default-number distributions will take place and be released on the NUS-CRI website. DCs are generated through a common factor model coupled with sparsely correlated residuals on one-month probabilities of default and other corporate exits. Through time aggregation and re-calibration, the DC model is used to produce various portfolio default-number distributions. Inclusion of DCs among corporate entities makes default-number distributions far more right-skewed, reflecting a much higher chance that entities in a typical credit portfolio may default together due to their reactions to a common shock such as commodity price, interest rate, exchange rate, etc. Users can track and monitor default-number distributions for their portfolios of interest over time.

CONTENT

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I. OVERVIEW

Default correlations (DCs) are crucial information for many practical applications that involve more than one obligor. Credit derivatives such as CDS, for example, stipulate payment obligation by the protection seller in an event that the reference obligor defaults. When called upon, the protection seller may also default. Such a double default situation cannot be adequately analyzed with marginal default probabilities alone. In general, DCs influence the default-rate (or default-number) distribution of a credit portfolio and their inclusion will make typical default-rate (or default-number) distributions far more right-skewed, reflecting a much higher chance that corporate entities in a typical credit portfolio may default together due to their reactions to a common shock such as commodity price, interest rate, exchange rate, etc.

The NUS-CRI approach to DCs comprises four steps: (1) identify a set of predetermined credit risk factors, estimate the factor model, and produce the factor model residuals; (2) estimate the time series dynamics of the predetermined credit risk factors and individual factor model residuals; (3) construct a sparse correlation matrix for the factor model residuals after taking out their individual time series effect; (4) further calibrate the model to the term structure of PDs at the time of application to take advantage of the information embedded in longer-term PDs.

Credit portfolios available on the NUS-CRI website contain aggregations of entities in 7 geographic regions, 133 economies, and 11/12 sectors (depending on choosing the NUS-CRI 2007 or NUS-CRI 2020 industry classification[1]). In each figure, each bar represents the probability of a certain number of companies from the selected group will default for the selected date and forecast horizon. Default-number and default-rate distributions are interchangeable given the total number of companies at the assessment time, and the NUS-CRI website only releases default-number distributions.

[1] Default-number distributions are provided for two sets of industry classification standards, namely, NUS-CRI 2007 and NUS-CRI 2020 industry classification standards. Users can choose the distribution using either classification standard.

II. METHODOLOGY

Default-rate (or default-number) distributions are generated using a bottom-up approach, which relies on modeling DCs through using the time series of one-month PDs and POEs for individual companies. Calculation of DCs by NUS-CRI is largely based on the methodology of Duan and Miao (2016)[2] except for factor model selection and part of re-calibration. The NUS-CRI procedure comprises[3]:

Step 1: Generating the common factor model

The first step is to construct and estimate the common factor model by using the logit-transformed PDs and POEs.[4] For individual firms, each one is expected to respond to common credit risk factors in a different manner, and the channels of influence can be identified by regressing each firm's logit-transformed one-month PDs and POEs on the common factors.

It is natural to expect that a firm's likelihood of default being influenced by the global, industry-specific and econ-specific credit risk factors. Thus, credit cycle indices (CCIs) are created by taking the logit-transformed global, industry and economy median PDs and POEs as credit factors. The way of constructing CCIs is akin to creating stock market indices except that medians are used instead of means. To be more relevant to the economic environment, other factors, including FX rate and interest rate, will also be considered. In total, there are around 420 potential common factors[5] to select from every month.

The firms are divided into 19 groups based on their industry sectors. Then factor selection is performed to select at most 15 common factors for each group using the modern zero-norm variable selection technique of Duan (2019).[6] The selected factors will be applied to the common factor model of each group.

[2] Duan, J.C., W. Miao, 2016, Default Correlations and Large-Portfolio Credit Analysis. *Journal of Business and Economic Statistics* 34, 51-65.

[3] For more technical details of the four steps, please refer to Addendum 3 to the CRI Technical Report (Version: 2021, Update 1).

[4] PDs and POEs are naturally bounded between 0 and 1. The logit transformation converts them to values in the whole real line and facilitates statistical modeling.

[5] The 420 potential common factors include 282 PDs and POEs CCIs (141 for PDs and POEs respectively, which include 1 global factor, 11 industry factors and 129 economy factors) and 138 other factors, which include 65 FX cycles and 73 interest rate cycles. The total number of potential factors may change every month due to data availability.

[6] Duan, J.C., 2019, Variable Selection with Big Data based on Zero Norm and via Sequential Monte Carlo. *National University of Singapore Working Paper*.

Step 2: Considering time dynamics

The next step is to estimate time dynamics for both factors and individual factor model residuals using VAR (vector auto-regression) and AR (auto-regression) models. A first-order AR model is used to account for the autocorrelation in the residual dynamics.

For global PD-CCI and POE-CCI pair, a first-order two-dimensional vector autoregression (VAR) is used to capture their joint time dynamics. For other factors including industry/economy CCIs which have been pairwise orthogonalized to the global pair, FX rates and interest rates, a first-order AR model is individually deployed. Together, we refer to them as the factor dynamics model.

Step 3: Constructing the sparse residual correlation matrix

To better capture co-movements of credit risks across firms, we recognize that DCs may go above and beyond common risk factors and be present in localized clusters due to credit and/or supply-chain relationships. Supplementary information on co-movements from the PD and POE residuals should therefore be extracted. We stack together all the error terms from their residual AR models to form a data matrix. Having employed the common risk factors, we have reasons to assume uncorrelatedness among most pairs of the AR model error terms, which should in principle result in a sparse correlation matrix. Due to missing data, only pairwise correlations are available, and statistical significance is a simple way to filter out insignificant pairwise correlations. The SCAD-thresholding element-wise method is then applied to achieve both sparsity and positive semi-definiteness of the correlation matrix.

Step 4: Calibrating to PD term structures

Up until this point, only historical time series of one-month PDs and POEs have been utilized in the estimation of the factor dynamics model with sparsely correlated residuals. Since term structures of PDs are available in the NUS-CRI database, further calibration of the factor dynamics model to term structures of PDs can take advantage of the additional and timely information embedded in them.

The common factor model, the factor dynamics, the residuals dynamics model, and the sparse residual correlation matrix can be combined to simulate future paths of the one-month PDs and POEs for any group of obligors over any horizon of interest. With one set of simulated paths in place, one can compute by the standard survival/default formula the randomized individual default probabilities conditional on each of the many paths. These conditional individual term structure of default probabilities can then be averaged over many simulations to arrive at PDs for different horizons and obligors, which should in principle match up with their observed term structure of PDs. In reality, model misspecification and sampling errors will prevent two sets to exactly match. To reduce the mismatch, re-calibration of the individual residuals dynamics under the factor dynamics model has been implemented as a minimization problem to increase the match.

III. DEFAULT-RATE DISTRIBUTIONS

How is the above methodology used in portfolio credit analysis? Especially how the default-rate distributions with DCs are generated? The answers are provided below, which also sheds light on the application of the methodology.

To conduct portfolio credit analysis, the bottom-up approach should be used. Individual obligors' default probabilities should first be calculated. The methodology introduced in section II helps to simulate future paths of one-month PDs and POEs for any target group of obligors to any future time point of interest. As a consequence, multiperiod default probabilities deduced from these random paths exhibit default correlations through co-movements induced by the common factors and various clusters of local correlations. Individual obligors' multiperiod default probabilities, exposures at default and recovery rates, conditional on the random paths, can then be aggregated to the portfolio level to compute various quantities of interest.[7] Averaging the conditional outcomes over many simulations then yields the final results for portfolio credit analysis.

For the sole purpose of generating default-rate distributions, exposures and recovery rates are not needed. A default-rate distribution can also be understood as equivalent to default-number distribution provided the number of obligors at the time of assessment is known.

Figure illustrates the convolution method that is used to compute default-number distributions. The reason is that conditional on a simulated path, defaults of different obligors are independent by construction.

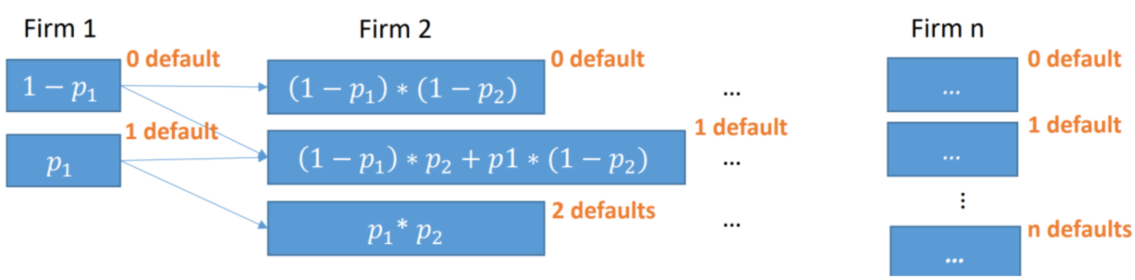

Figure 1: Convolution Method

The numerical execution of the above convolution operation can utilize a simple expanding vector for recording the default-number distribution up to, say, *k* obligors.

[7] For a technical description of the two types of conditional aggregation algorithms of Duan and Miao (2016), please refer to the reference given in Footnote 2.

Adding an obligor taken from the remaining pool simply requires to expand the vector by one element to house the default-number distribution for up to *k*+1 obligors. Simple revision to the probabilities as in Figure 1 is the only needed step. In fact, the vector expansion will quicky cease to be necessary because the probability of exceedance for many concurrent defaults becomes negligible.

The above convolution method can generate the default-number distribution conditional on a simulated path governed by the factor dynamics model coupled with sparse residual correlations. After repeating the simulation-convolution procedure, say, 1,000 times, there are 1,000 copies of the conditional default-number distributions. The final default-number distribution is then obtained by calculating the Monte Carlo estimate of the distribution, which is the average of the 1,000 default-number distributions. Dividing the various potential default numbers by the number of obligors at the time of assessment, the default-number distribution becomes a default-rate distribution.

IV. APPLICATION

Figures 2 and 3 show the impact of incorporating DCs into default-rate distributions. Each of these graphs show default-rate distributions with and without DCs for one of three regions under one of two future horizons, namely global, United States, and China over 3 and 12 months in Mar-2022. Evident through these graphs, incorporation of DCs tends to make the default-rate distribution much more right-skewed, reflecting a general tendency for firms in the same economy/sector to co-move as far as credit risk is concerned. The long right tail in these graphs suggests that ignoring DCs can cause a severe under-assessment of the chance for joint defaults to occur, a real threat facing a credit portfolio. The same conclusion holds regardless of the horizon being short or long.

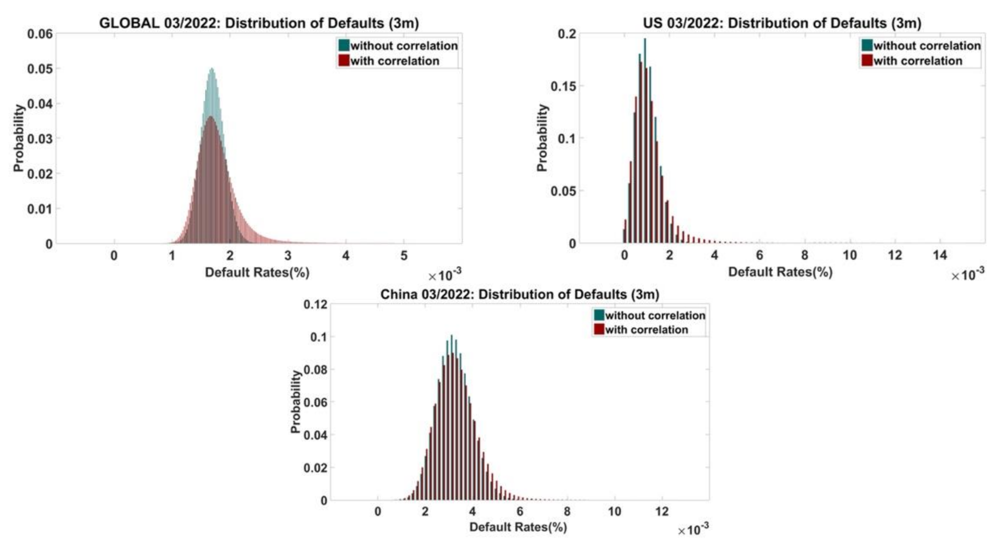

Figure 2: Correlated and non-correlated default-rate distributions for the global economy, US, and China (3-month horizon)

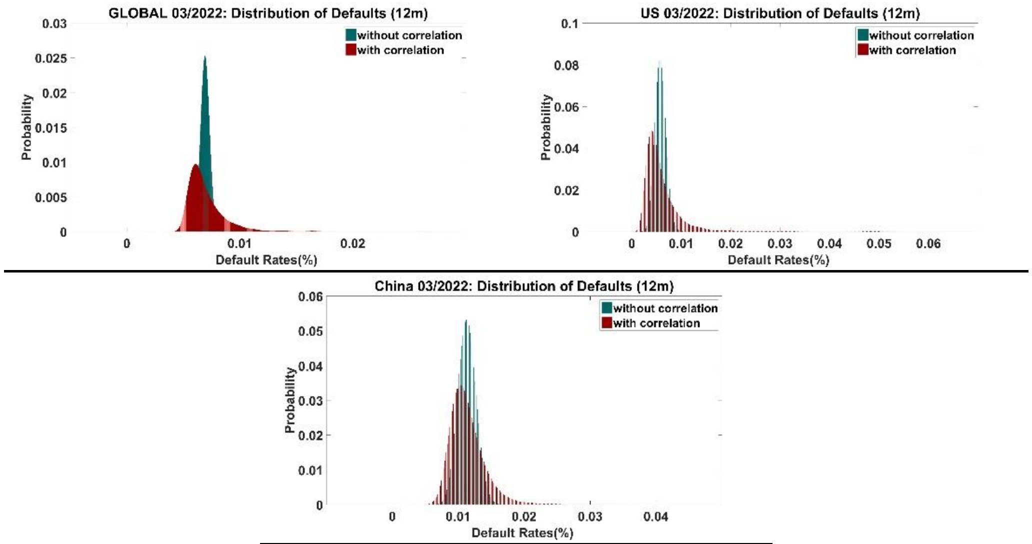

Figure 3: Correlated and non-correlated default-rate distributions for the global economy, US, and China (12-month horizon)

Default-rate distributions with DCs are sensitive to the prevailing economic condition. Figure for the 12-month horizon demonstrates how the default-rate distribution reacts to the initial onset of the COVID-19 pandemic in March 2020 vs two years later in March 2022 when the pandemic-propelled stimuli around the world started tapering.

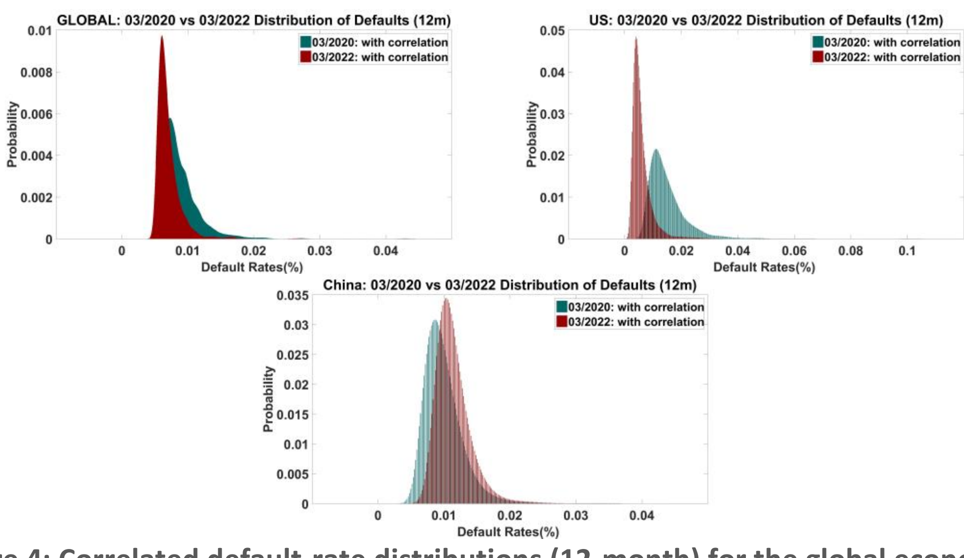

Figure 4: Correlated default-rate distributions (12-month) for the global economy, US and China in Mar-2020 (COVID-19 shock) vs Mar-2022 (tapering)

Figure 4 clearly shows that the default-rate distributions for both the global economy and the US were more right-skewed in 2020 as compared to 2022, signaling that the two economies in 2022 were in a better shape as compared to Mar-2020 during the initial onset of the COVID pandemic. Interestingly, the two default-rate distributions for China suggest that the economy was marginally worse-off in 2022 compared to 2020, possibly due to the slowdown of real economic activity in the country as new COVID-19 related lockdown measures took effect and the lingering Evergrande crisis impacting financing access for vulnerable firms in China.

SUMMARY

The incorporation of DCs tends to make the default-rate distribution much more right-skewed, reflecting a general tendency for firms in the same economy/sector to comove as far as credit risk is concerned. This can be useful for regulatory authorities, central banks, and commercial/investment banks for increased awareness and understanding of credit risks at the portfolio level, since ignoring DCs can cause a severe under-assessment of the chance for joint defaults to occur, a real threat facing a credit portfolio.

DC provides a low-rank factor model with sparsely correlated residuals to model short-term PDs and POEs. This factor model is meant to effectively handle high dimensionality inherent in default correlations for a large number of obligors, and serves to generate default correlations through correlated short-term PDs and POEs. The common factors employed for the low-rank part are some intuitive predetermined global, industry and economy credit risk factors by aggregating PDs and POEs into CCIs, and other macro-economic factors including FX and interest rate cycles. The distribution of defaults with correlation for the portfolio of interest can then be constructed by the convolution algorithm.

V. REFERENCES

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ABOUT THE CREDIT RESEARCH INITIATIVE

The Credit Research Initiative (CRI) was launched by Professor Jin-Chuan Duan in July 2009 at the Risk Management Institute of the National University of Singapore. CRI has since shifted to the Asian Institute of Digital Finance (AIDF) in 2021. Aiming at "Transforming Big Data into Smart Data", CRI covers over 85,000 public firms and produces daily updated Probabilities of Default (1-month to 5-year horizon), Actuarial Spreads (1-year to 5-year contract) and Probability of Default implied Ratings on over 45,000 currently active, exchange-listed firms in 134 economies. CRI also distributes historical time series of over 40,000 inactive firms due to bankruptcy, corporate consolidation or delisting for other reasons. In addition, CRI produces and maintains Corporate Vulnerability Indices (CVI), which can be viewed as stress indicators, measuring credit risk in economies, regions, and special portfolios.

As a further step, CRI converts smart data to actionable data to offer bespoke solutions to meet demands of its users. A concrete example is our development of the BuDA (Bottom-up Default Analysis) toolkit in collaboration with the International Monetary Fund (IMF). BuDA is an automated analytic tool based on the CRI-PD system, enabling IMF economists to conduct scenario analyses on the macroeconomic and financial linkage.

CRI publishes Weekly Credit Brief and Semi-Annual Credit Summary, highlighting key credit-related events, offering insights based on the CRI-PDs of the entities involved, and providing useful statistics on credit risk of economies and/or sectors.

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