PROBABILITY OF DEFAULT WHITE PAPER

Credit Research Initiative of the National University of Singapore

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ABSTRACT

Probability of Default (PD) is the core credit product of the Credit Research Initiative (CRI) default prediction system. The system is built on the forward intensity model developed by Duan *et al.* (2012, *Journal of Econometrics*). This white paper describes the fundamental principles and the general mechanics of the model. Details of the theoretical foundations and numerical realization are presented in RMI-CRI Technical Report (Version 2017 Update 1). This white paper contains three sections. Sections One and Two describe the methodology and performance of the model respectively, section Three relates to the competitiveness of the CRI PD.

CONTENT

OVERVIEW

Probability of Default (PD) is the main credit product of the CRI default prediction system built on the forward intensity model by Duan *et al.* (2012, Journal of Econometrics). This forward intensity model is governed by two independent doubly stochastic Poisson processes, operating on forward time instead of spot time. This enables the model to produce forward-looking PD-term structures of firms based on the dynamic learning from the credit-related macroeconomic and firm-specific data.

The key features of this model:

- A combination of reduced-form model (based on a forward intensity construction) and structural model (using the distance-to-default as one of its input covariates)
- Accommodate the two risks that a firm might encounter; namely default risk and risks of other types of exits (i.e. merger and acquisitions)
- Use forward probabilities of default and other types of exits as building blocks to construct the PD-term structure in a consistent manner
- Employ twelve input covariates (default predictors) of both market-based and accounting-based firm-specific attributes, as well as the macroeconomic factors

In July 2010, CRI began to release daily updated PD of around 17,000 listed firms in 12 Asian economies. As of February 2017, the CRI PD coverage has expanded to about 65,000 exchange-listed firms in 120 economies with the prediction horizons from 1 month to 5 years. Out of those firms, about 33,000 are currently active and have their PD updated on a daily basis.

METHODOLOGY

MODEL DESCRIPTION

The CRI default prediction system based on the forward intensity model is an engine for corporate default prediction which offers a practical bottom-up approach to aggregate individual firms' behaviors into a portfolio's default profile. In reality, a firm can exit a stock exchange due to default or other forms of exit such as mergers and acquisitions. Therefore, the CRI default prediction model accommodates these two competing risks by modelling the occurrences of default and other exit as two independent Poisson processes, each with its own stochastic intensity. Forward intensities are the building blocks to generate the forward-looking PD term structure from one month up to five years. Figure 1 demonstrates the infrastructure of forward probability – the probability of a firm surviving for 3 months from today and default in the next month.

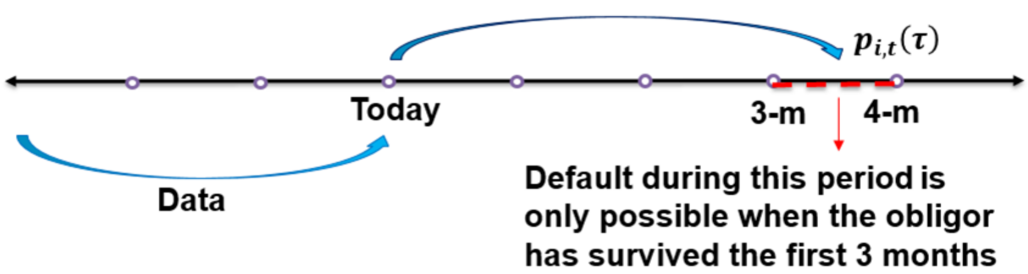

Figure 1: Forward Probability in the CRI model

Default probability of firm i at time t for each forward period τ, denoted by $p_{i,t}(\tau)$, is captured by a function of input covariates that characterize the state of the economy (macroeconomic risk factors X_t) and the vulnerability of individual obligors (firm-specific attributes $Y_{i,t}$):

$$p_{i,t}(\tau) = P_\tau(X_t, Y_{i,t})$$

The underlying forward intensity functions are parameterized in order to compute the forward-looking PD. The parameters are estimated during the model calibration which is performed on a monthly basis to ensure that the model remains current and organic.

INPUT COVARIATES

Following the notation above, firm i's input covariates at time t are represented by 1) the vector X_{t} that is common to all firms in the same economy[1], and 2) a firm-specific vector Y_t with components constructed from the firm's financial statement and market capitalizations. The CRI default prediction model employs two macroeconomic variables and ten firm-specific variables, described in Table 1 below.

Table 1: Input Covariates employed by the CRI model

	Model Inputs	Description
Macro-Economic Factors	Stock Index Return	Trailing 1-year return of the prime stock market
	Short-term Risk-Free Rate	Yield on 3-month government bills
Firm-Specific Attributes	Distance-to-Default (level)	Volatility-adjusted leverage based on Merton (1974) with special treatments
	Distance-to-Default (trend)	
	Cash/Total Asset (level)	Liquidity - Ratio of each firm's sum of cash and short-term investments to total assets
	Cash/Total Asset (trend)	
	Net Income/Total Asset (level)	Profitability - Ratio of each firm's net income to total assets
	Net Income/Total Asset (trend)	
	Relative Size (level)	Logarithm of the ratio of each firms market capitalization to the economy's median market capitalization
	Relative Size (trend)	
	Market-to-Book Ratio	Market misvaluation/ Future growth opportunities
	Idiosyncratic Volatility	1-year idiosyncratic volatility of each firm, computed as the standard deviation of its residuals using the market model

[1] Firms whose are listed on the stock exchanges of that economy

In the table above, “level” is computed as the 12-month moving average (A minimum of six observations in the 12-month range are required, otherwise level variables will bear missing values.), and “trend” is computed as the current value minus the “level” value(If the current month value is missing, the trend variable is set to be the last valid value during the previous). The “trend” measure captures the momentum effect and gives a hint about the direction of future movements. Duan *et al.* (2012) shows that using the level and trend of the measures for some input covariates significantly improves the predictive power of the model, particularly for short-term horizons.

In order to understand the momentum effect, consider the case of two firms that have the same current value of Distant-to-Default (DTD). Firm 1 reaches its current value of DTD from a lower level, while Firm 2 reaches the same current value of DTD as Firm 1 but from a higher level (*see Figure 2*). If only the current value of the DTD is employed for default prediction, the impact of the DTD on the PD would be identical for both firms. However, intuitively, one would expect that the DTD of Firm 1 would keep increasing and that the DTD of Firm 2 would continue to decrease. In order to account for such momentum effects, CRI uses both level and trend attributes in its PD calculations.

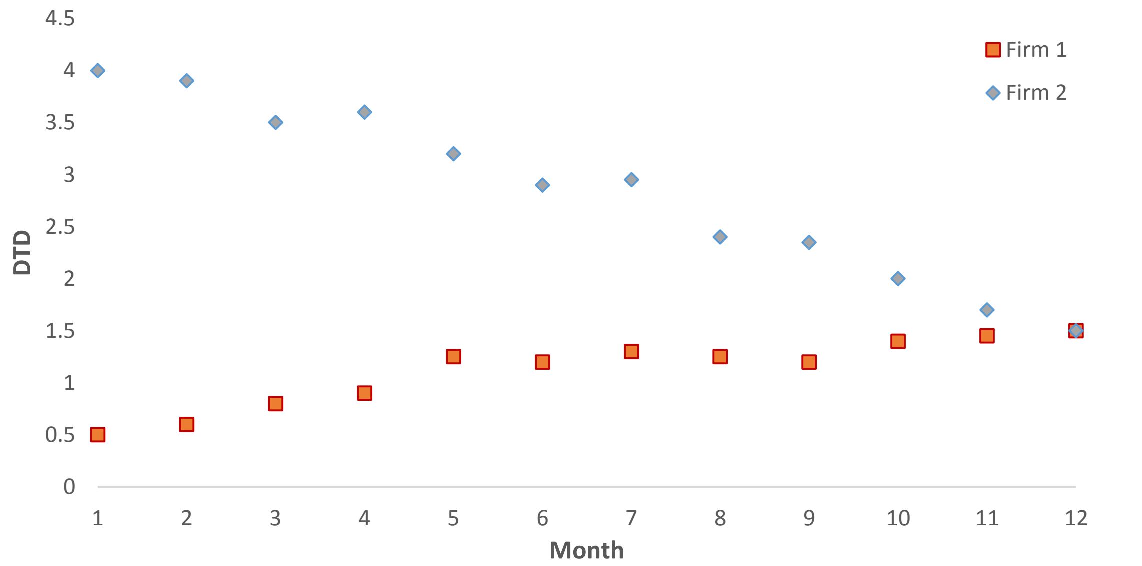

Figure 2: Two firms with equal current value, but DTD trending in the opposite direction.

DTD has long been recognized as an important indicator of a firm’s credit quality, and is employed by CRI as a default predictor in the forward intensity model. Typically, for each firm, DTD is estimated using a Merton-based structural default prediction model with KMV model assumptions on the debt maturity and size, i.e.,

$$DTD_t = \frac{\log\left(\frac{V_t}{L}\right) + \left(\mu - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

where V_t is the asset value following a geometric Brownian motion with drift μ and volatility σ, L is the default point with value equal to short-term liability plus half of long-term liability, and $\sqrt{T-t}$ is set to 1 year.

However, to improve the traditional DTD measure, CRI implements some special treatments on its own DTD calculation to oversome some drawbacks that have been identified in the literature.

The key treatments are:

- Follow Duan (2010) to add a fraction (δ) of other liability to the KMV default point L
- Set $\mu = \frac{\sigma^2}{2}$ to improve the stability of estimation.
- Standardize the firm's market value by its book value to well-handle the scale change due to a major investment and financing action

The DTD parameters are estimated by maximum likelihood method described in Duan (1994, 2000).

A brief expression of the CRI's version of DTD is

$$DTD_t = \frac{\log\left(\frac{V_t}{L}\right)}{\sigma\sqrt{T-t}}$$

where the default point is set to

$$L = \text{Current Liability} + \frac{1}{2}\text{Longterm Liability} + \delta \times \text{Other Liability},$$

and $\delta \in [0,1]$ are specified and estimated for sectors in each calibration group.

CALIBRATION

Currently the data for the CRI default prediction system comes from various international data distributors. It is worthwhile to note that there is little to no credit default events in certain economies due to limited number of listed firms, which means that the calibration of models for individual economy would not be statistically meaningful. In view of this, firms around the world are categorized into six calibration groups according to certain similarities in the stage of economic development and geographic locations of their listed exchanges. Theses calibration groups are North America, Europe, Asia-developed economies, Emerging Markets, China and India.

The CRI PD of firms in the same calibration group share the same set of parameters, (except for some covariates in some special circumstances). In order to overcome the difficulties in optimization that are caused by the high dimensionalities of parameters, CRI uses the sequential Monte Carlo method for its PD model estimation. Details of the procedure can be found in the RMI-CRI Technical Report (2017).

MODEL PERFORMANCE

Accuracy Ratio is one of the popular quantitative measures for evaluating the discriminatory power of a default prediction system. It is the ratio of the differential of the performance of the evaluated system and the random system over the differential of the performance of the perfect system and the random system. The interpretation of AR is that if defaulted firms have been assigned among the highest PD before they defaulted, then the model has discriminated properly between the safe and risky firms. The CRI default prediction system achieves high AR scores for all its covered regions and economies, indicating its good performance. Figure 3 below illustrates the AR of the CRI default prediction system for the United States for horizons from 1 month to 5 years.

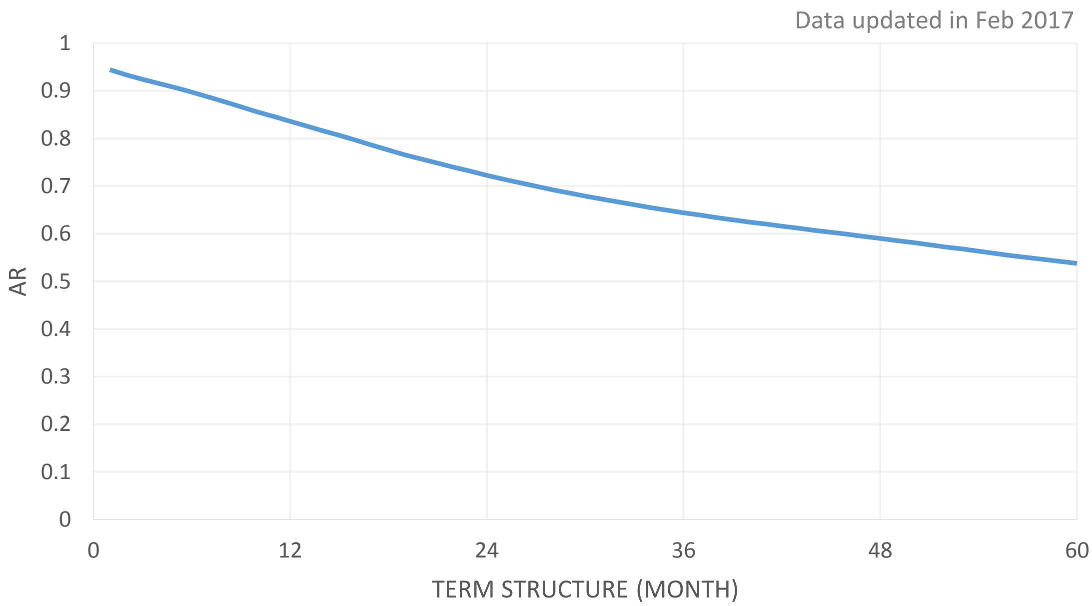

Figure 3: Accuracy Ratio of the United States in the CRI PD model

Figure 4 depicts the aggregated CRI PD for the US, the financial sector in US, Singapore and Thailand. One can notice that the effects of 1997 Asian financial crisis, the dot-com bubble in 2000 and the global financial crisis in 2007-2008 are well captured by the PD movements.

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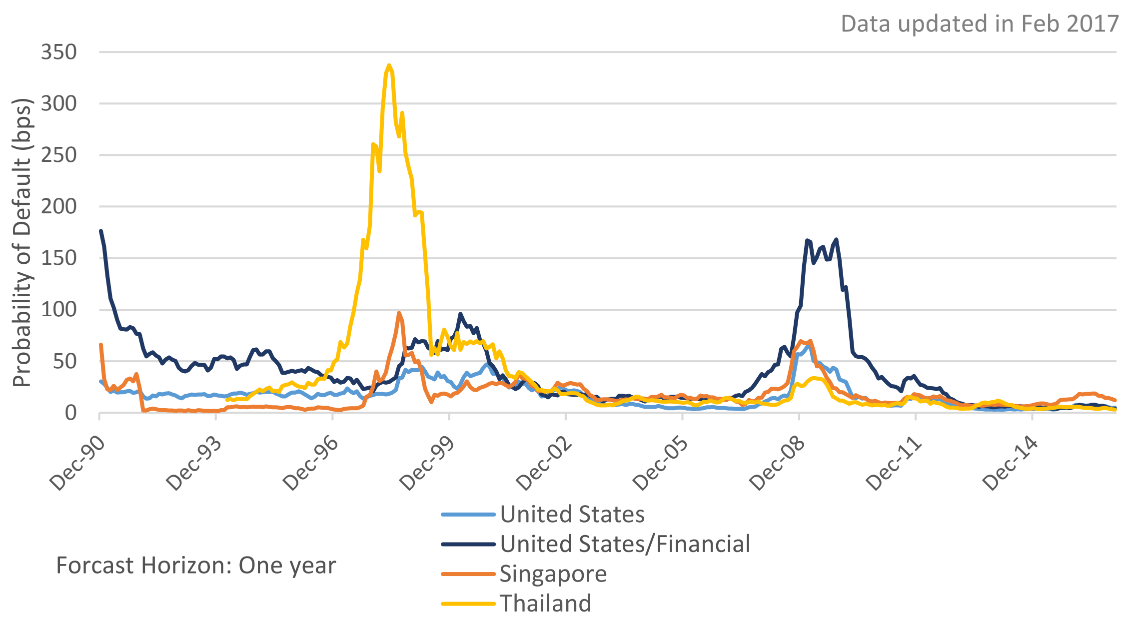


Figure 4: Historical time series of Aggregated CRI PD

The Figure 5 below reports the realized number of defaults compared to the predicted number of defaults over 1 year for North America.

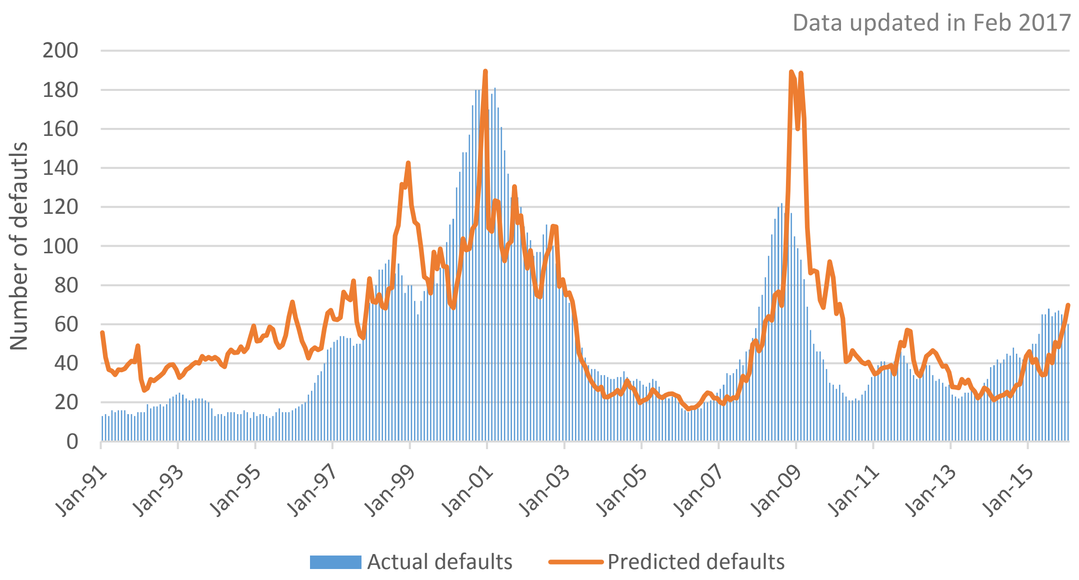
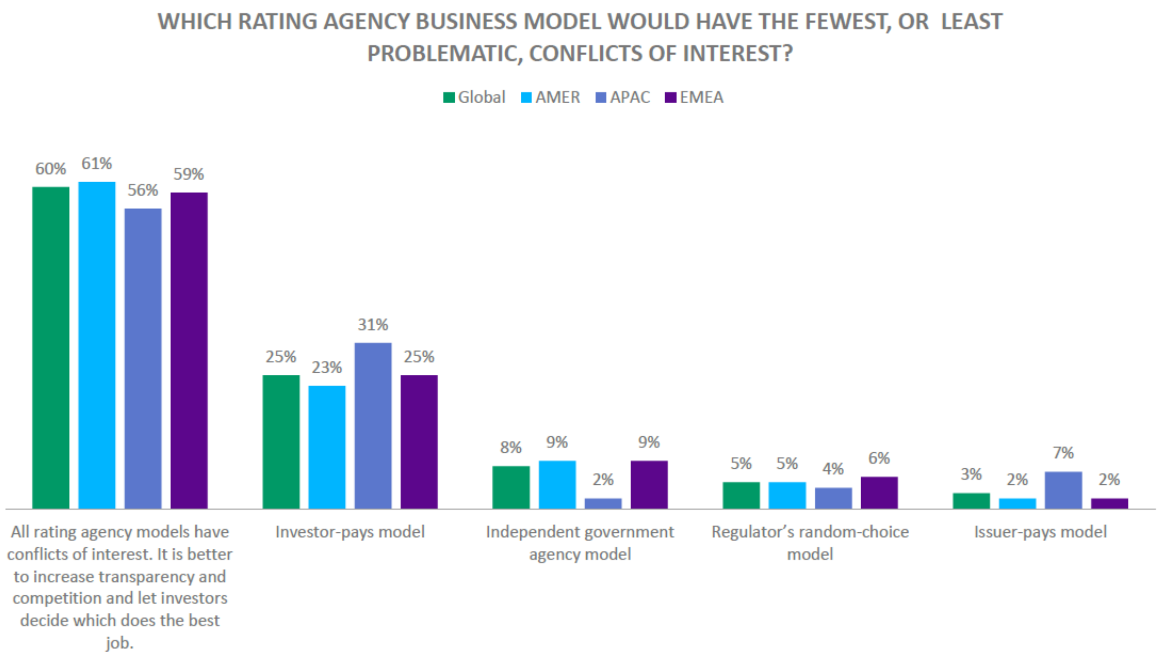


Figure 5: Realized number of defaults vs. predicted number of defaults for North America

COMPETITIVENESS

Traditionally, "sell-side" credit rating is structured on the issuer-pay principle. From a public interest point of view, it has been demonstrated that this could be a seriously flawed business model with potential "moral hazard" and "rating shopping" problems. The Chartered Financial Analyst (CFA) Institute conducted a survey in 2008 in which 11% of the 1,946 respondents claimed that they have personally witnessed credit rating change due to external pressure. Another survey conducted by the CFA Institute in 2014, reported that 60% of the respondents think all rating agency models have shortcomings, and that they believe increasing transparency and competition is the best solution to overcome those issues (*see Figure 6*). By using a methodology that is recognized by the scientific community, CRI promotes sound credit risk models and contributes to making credit rating methodologies more transparent. Furthermore, our credit rating model implementation is free of ad-hoc human discretions, apart from dealing with occasional data errors that would be expected from time to time.



Source: CFA Institute 2014 credit rating agency survey report.

Figure 6: Changing the issuer-pay model and increased transparency around the way ratings are established are needed to improve reliability of CRAs

The CRI PD also allows the enhancement of the granularity of credit information. Although PDiR is provided as a tool to compare those PD with the ratings of other credit rating agencies, it is strongly recommended to not only use letter ratings but also take the more precise PD numbers into consideration. Another problem with letter credit ratings is that they are often determined vaguely on a short-term or long-term basis. Alternatively, CRI strives to delivery a precise term structure of PD. As an example, Figures 7a and 7b demonstrate Lehman Brothers' risk profile with forward-looking default probability term structure ranging from 1 month to 60 months.

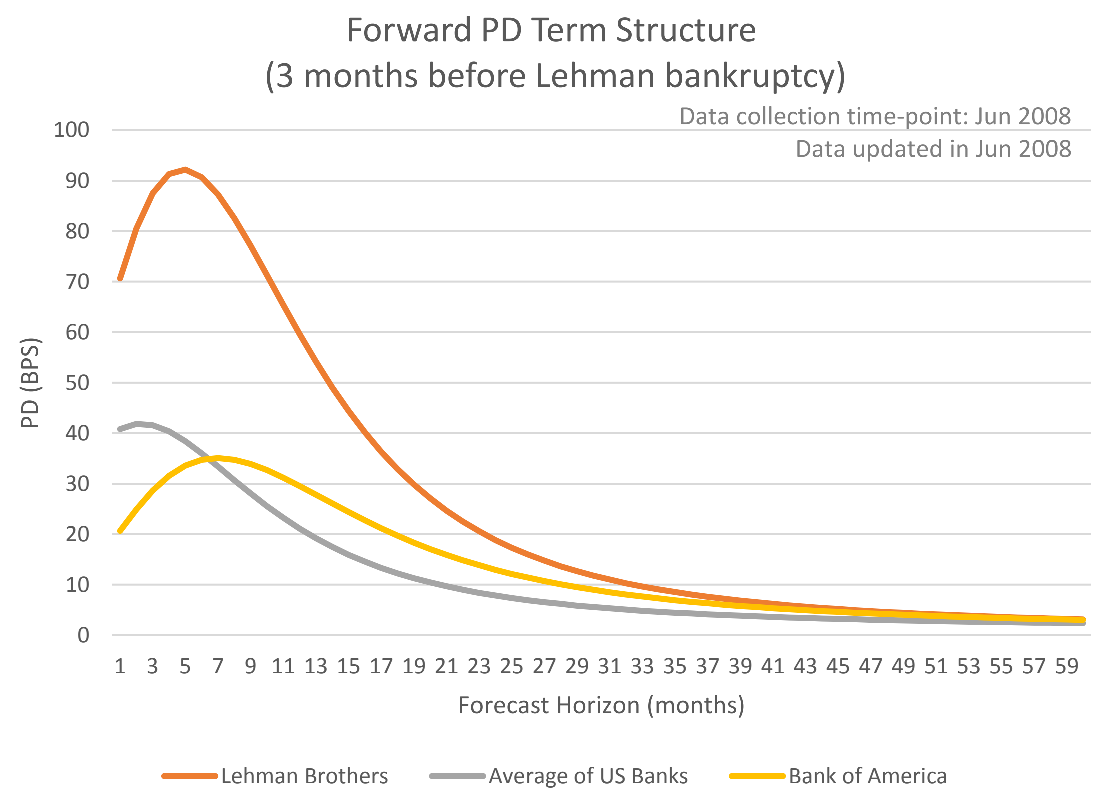


Figure 7a: Lehman Brothers' risk profile 3 months before bankruptcy

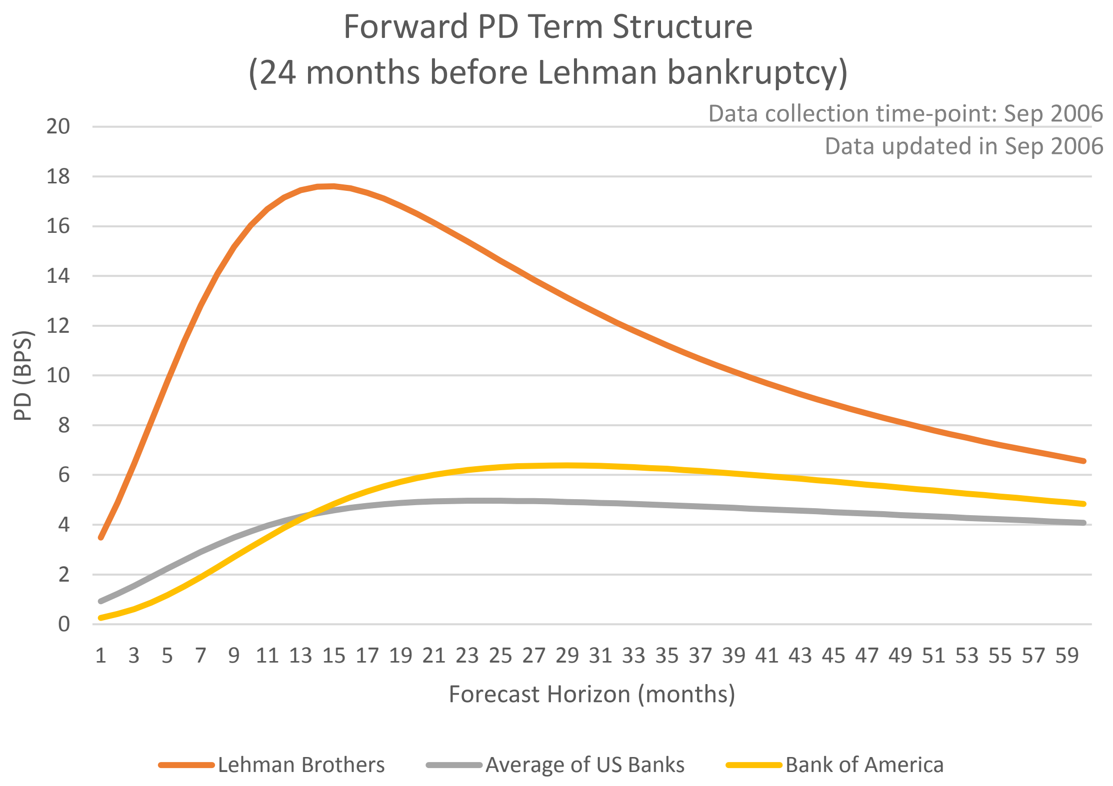


Figure 7b: Lehman Brothers' risk profile 24 months before bankruptcy

CONCLUSION

The CRI PD evaluates the default risk of public listed firms by quantitatively analyzing their financial statements, stock market data and macroeconomic factors retrieved from various international data distributors. Unlike credit models that utilize letter ratings, the CRI PD is a precise gauge for credit risk with term structure ranging from 1 month to 5 years. The CRI default prediction system yields high performance in terms of default prediction accuracy.

CRI currently provides daily updated PD for about 33,000 active and exchange-listed firms globally.

APPENDIX

The CRI PD Model - Forward and Cumulative Probabilities

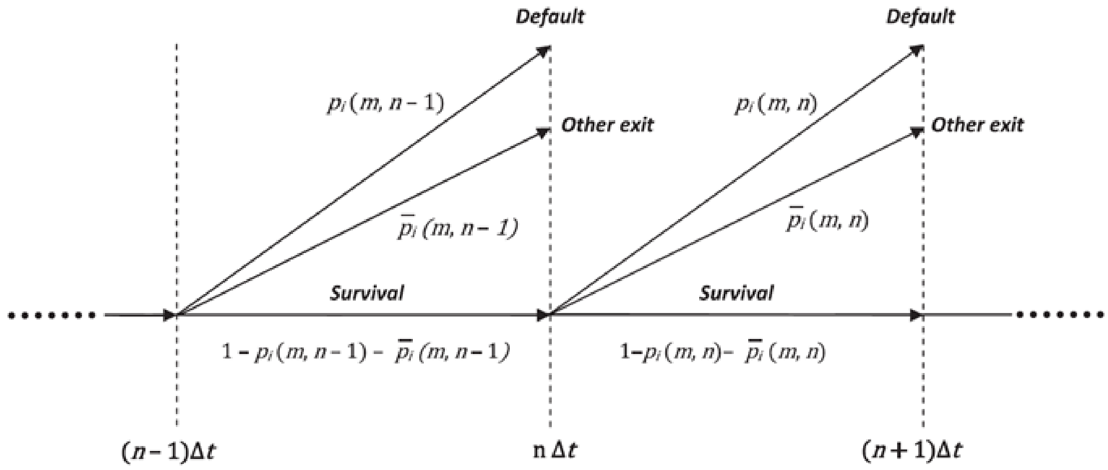


Figure 8: Forward Probability in the CRI model

Probability of Default: $p_i(m,n)$ is a conditional probability viewed from $t = m\Delta t$ that firm i will default before $(n+1)\Delta t$, conditioned on firm i surviving up until $n\Delta t$.

Probability of Other Exit: $\bar{p}_i(m,n)$ is a conditional probability viewed from $t = m\Delta t$ that firm i will exit before $(n+1)\Delta t$, conditioned on firm i surviving up until $n\Delta t$.

Probability of Survival: $1 - p_i(m,n) - \bar{p}_i(m,n)$, as default, other exit and survival are mutually exclusive events.

A particular path for firm i, predicting at time $t = m\Delta t$, survives till $(n-1)\Delta t$ and defaults between $(n-1)\Delta t$ and $\mathrm{n}\Delta\mathrm{t}$, with τ_i and $\bar{\tau}_i$ represent default and other-exit time, respectively:

$$\mathrm{Prob}_{t=m\Delta t}[\tau_i = n, \tau_i < \bar{\tau}_i]$$
$$= p_i(m, n-1) \prod_{j=m}^{n-2} [1 - p_i(m,j) - \bar{p}_i(m,j)]$$

From that we can derive the cumulative default probability indicating the default probability within certain period:

$$\mathrm{Prob}_{t=m\Delta t}[m < \tau_i \le n, \tau_i < \bar{\tau}_i] = \sum_{k=m}^{n-1} \left\{ p_i(m,k) \prod_{j=m}^{k-1} [1 - p_i(m,j) - \bar{p}_i(m,j)] \right\} \qquad Eq. (1)$$

The forward intensity for the default of firm i that is observed at time $t = m\Delta t$ for the forward time interval from $t = n\Delta t$ to $(n+1)\Delta t$ is denoted by $h_i(m,n)$ where $m \le n$. The corresponding forward intensity for a non-default exit is denoted by $\bar{h}_i(m,n)$. As defaults are signalled by a jump in a Poisson process, its conditional probability is a simple function of its forward intensity:

$$p_i(m,n) = 1 - \exp[-\Delta t \, h_i(m,n)]$$

Since joint jumps in the same interval are assigned as defaults, the conditional other exit probability needs to take this into account:

$$\bar{p}_i(m,n) = \exp[-\Delta t \, h_i(m,n)] \times \{1 - \exp[-\Delta t \, \bar{h}_i(m,n)]\}$$

It remains to be specified on the dependence of the forward intensities on the input covariate $X_i(m)$:

$$h_i(m,n) = \exp[\beta(n-m) \cdot \, Y_i(m)]$$

$$\bar{h}_i(m,n) = \exp[\bar{\beta}(n-m) \cdot \, Y_i(m)]$$

Where, β and $\bar{\beta}$ are coefficient vectors that are functions of the number of months between the observation date and the beginning of the forward period $n-m$, and $Y_i(m)$ is the vector $X_i(m)$ augmented by a preceding unit element: $Y_i(m) = (1, X_i(m))$.

A notion H is introduced for the forward intensities so that it becomes clear which parameters the forward intensity depends on:

$$H(\beta(n-m), X_i(m)) = \exp[\beta(n-m) \cdot \, Y_i(m)]$$

The cumulative default probability given in Eq. (1) in terms of the forward intensities is then:

$$
\begin{aligned}
&\mathrm{Prob}_{t=m\Delta t}[m < \tau_i \leq n, \tau_i < \bar{\tau}_i] \\
&= \sum_{k=m}^{n-1} \left\{ \{1 - \exp[-\Delta t\, H(\beta(k-m), X_i(m))]\} \right. \\
&\qquad \times \exp\left\{ -\Delta t \sum_{j=m}^{k-1} [H(\beta(j-m), X_i(m)) \right. \\
&\qquad \left.\left. +\ H\left(\bar{\beta}(j-m), X_i(m)\right)] \right\} \right\}
\end{aligned}
$$

REFERENCES

Duan, J. C., Sun, J., and Wang, T. (2012). "Multiperiod corporate default prediction – A forward intensity approach", *Journal of Econometrics*, 179, 191-209.

Duan, J. C. (2010). "Clustered Defaults". *Risk Management Institute Working Paper*.

Duan, J. C. (2000). "Correction: Maximum Likelihood Estimation Using Price Data of the Derivative Contract". *Mathematical Finance*, 10(4), 461-462.

Duan, J. C. (1994). "Maximum Likelihood Estimation Using Price Data Of The Derivative Contract". *Mathematical Finance*, 4(2), 155-167.

Merton, R. C. (1974). "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates". *The Journal of Finance, 29* (2), 449-470.

NUS Risk Management Institute (RMI) – Credit Research Initiative
Address: 21 Heng Mui Keng Terrace, I^3 Building, Level 4, Singapore 119613
Tel: (65) 6516 3380 Fax: (65) 6874 5430
Website: http://rmicri.org/