ACTUARIAL SPREAD WHITE PAPER

CONSTRUCTION AND APPLICATIONS OF THE ACTUARIAL SPREAD (JULY 2014)

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SUMMARY

In July 2014, the Risk Management Institute (RMI) at the National University of Singapore launched the Actuarial Spread. The Actuarial Spread provides a new perspective to credit risk and is an alternative measure using the RMI PD. It provides an intuitive communication tool in a metric that market participants are more familiar with. The RMI Probabilities of Default (RMI PDs)[1] of individual firms are used as the physical PD in the CDS computation. This actuarial spread is constructed as if it were a CDS premium rate for risk-neutral market participants and it can be computed with a term structure of physical PDs and recovery rate assumption.

Figure 1 demonstrates the utility of the Actuarial Spread as a credit risk measure. The two graphs on the top half of the figure are associated with Eastman Kodak at the end of December 2011, shortly before Kodak's default. The two graphs on the bottom half of the figures are associated with Fujifilm at the same time. The graphs on the left side present the cumulative probabilities of default (PDs) that RMI-CRI has been publishing since the start of our operations in July 2010. While it is clear from the two graphs on the left that Kodak has higher credit risk than Fujifilm, PDs are a

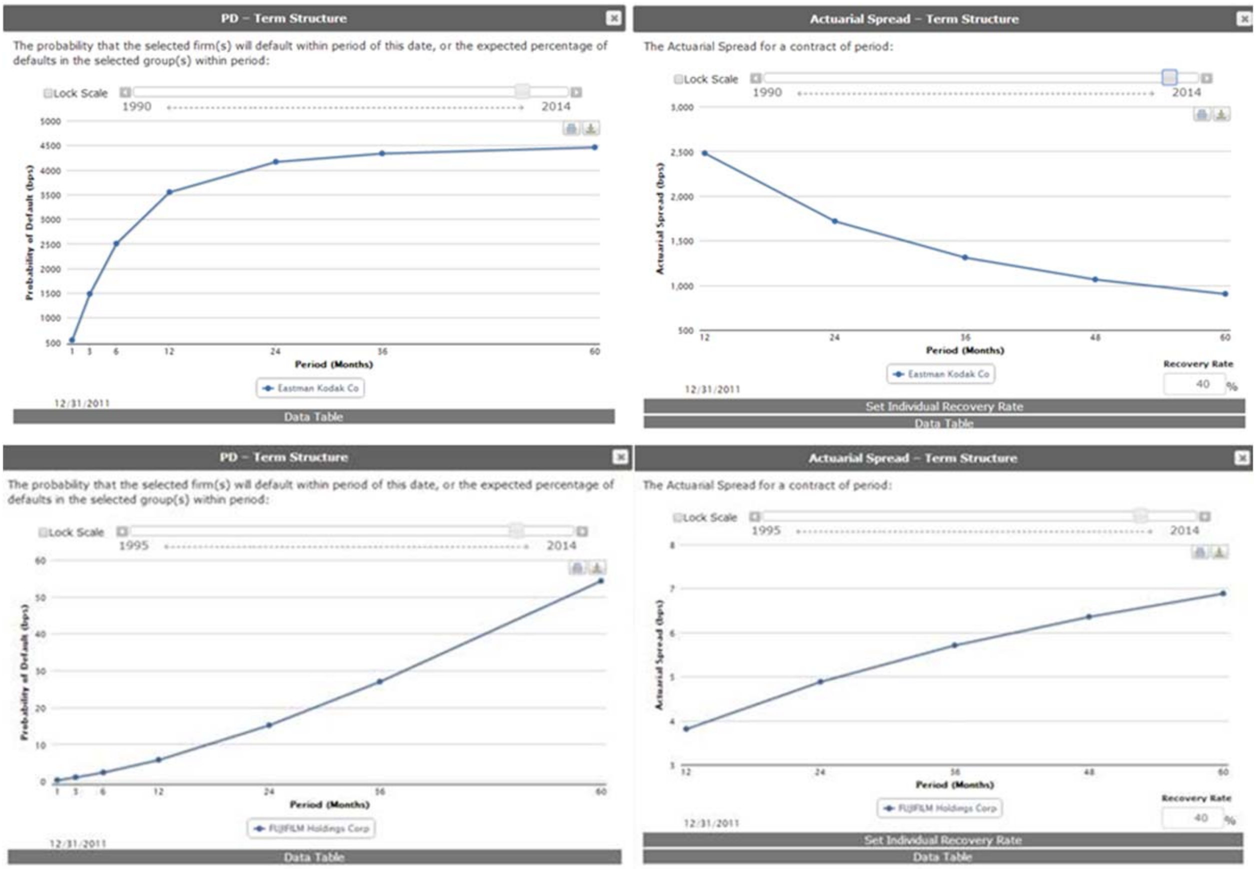

Figure 1. Term structure of probabilities of default for Kodak (top left) and Fuji (bottom left) in December 2011, and term structure of Actuarial Spread for Kodak (top right) and Fujifilm (bottom right).

[1] RMI PDs are the product of RMI's "public good" Credit Research Initiative conceptualized in March 2009 by the NUS RMI director, Professor Jin-Chuan Duan. Details on the RMI Credit Research Initiative are available at http://rmicri.org/.

metric which financial practitioners are less familiar with. On the other hand, the graphs on the right introduce the new Actuarial Spread launched by RMI-CRI in July 2014. As these are calculated similar to CDS par spreads, the values will be more familiar to financial practitioners.

Further applications of the Actuarial Spread are that its predictive relationship with real world CDS contracts can be a good basis for empirical pricing of CDS. Also, credit risk management systems based on Actuarial Spreads will be much more comprehensive than systems that use CDS spreads, as Actuarial Spreads cover a much larger set of firms.

In line with the Credit Research Initiative's philosophy as a "public good", putting the actuarial spreads into the public domain brings an unprecedented level of information availability and transparency to the field of corporate credit risk.

This white paper draws heavily from materials in Duan (2014) [2], where the theoretical foundations and numerical realization are presented in greater detail. Instead of rewording different parts of Duan (2014), we will conveniently take whole paragraphs from that paper without explicitly noting so by quotations.

CONSTRUCTION OF ACTUARIAL SPREAD

Actuarial spreads are calculated and reported daily for every public listed firm under the RMI-CRI coverage. As displayed in Figure 2 below, historical time series of the actuarial spreads are available, and the tenor of actuarial spreads ranges from a minimum of 1-year to a maximum of 5-years in one year increments.

Construction and calculation of the actuarial spread are based on the assumption that market participants are risk-neutral and the spread is such that no exchange of money is required initially. We can see it as equivalent to pricing CDS purely based on their actuarial values. This rate therefore will be referred to as an actuarial spread and can be computed using the term structure of physical PDs.

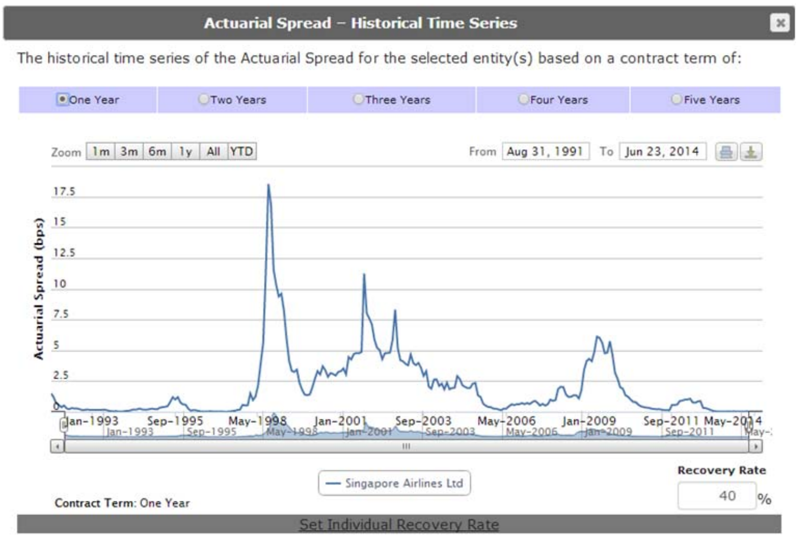

Figure 2. Actuarial spread for Singapore Airlines Ltd based on 1 year contract term (historical).

[2] Duan, J.-C. (2014), Actuarial Par Spread and Empirical Pricing of CDS by Decomposition. Global Credit Review, Vol. 4, 51-65.

But when PDs exhibit a term structure behavior, actuarial spreads become some complex aggregate of the PDs over the segment of the term structure defined by the tenor of CDS. Since the maximum loss given default is 100%, one would expect the CDS spread to be capped at 10,000 bps. However, in early 2012 Greek sovereign CDS were actually traded at a spread beyond 20,000 bps because of the risk of imminent default. This was because the protection seller was expected to soon incur a big loss whose amount was not tied to the accrual period. The actuarial spreads subject to daily accrual would need to be high enough to reflect the imminent default. As an explicit example, suppose an investor enters into a CDS as the protection seller with a par spread of 20,000 bps. Suppose further that the reference entity defaults two weeks later and the recovery rate is later determined to be 40%. The protection buyer is compensated for 6,000 bps, but must make a premium payment for the 2 weeks which will work out to less than 800 bps. Although the par spread was very high at 20,000 bps, the protection buyer would have come out ahead on this trade.

The actuarial spread formula can be readily computed daily by leveraging the RMI-CRI infrastructure, which allows free access by all registered users to daily updated PDs ranging from one month to five years for over 60,000 exchange-listed firms in 106 economies around the world. The current default prediction system used by RMI is based on the forward-intensity model of Duan, Sun and Wang (2012)[3] with parameters estimation based on Duan and Fulop (2013)[4]. The forward intensity functions used to generate the RMI-CRI PDs are exponential linear functions of some input variables (2 macroeconomic factors and 10 firm-specific attributes) where the coefficients depending on the forward starting time. The RMI-CRI model's parameters are re-calibrated monthly and the inputs to the functions are updated daily. For more details on this model implementation, please refer to the RMI-CRI Technical Report (2013)[5].

The remaining sections present more details on the construction of the actuarial spread.

TERM STRUCTURE

In the earlier years of CDS trading, the spread was generally set such that no upfront fee was required, and CDSs were quoted as a running par spread. In July 2009, fixed coupons were introduced as shown in Table 1.

Fixed coupon (bps)	EU	North and Latin America	Australia, Japan, New Zealand	Asia, Emerging Markets
25	Yes	No	Yes	No
100	Yes	Yes	Yes	Yes
500	Yes	Yes	Yes	Yes
1000	Yes	No	No	No

Table 1. Corporate CDS fixed coupons according to the ISDA standard.

The par spread of a CDS transaction rarely matches the fixed coupons; as a consequence the equivalent of the net present value of the difference between the actual spread and the coupon is paid up front. When the spread is based on the assumption that no up-front payment is made, it will be referred to as the CDS par spread and denoted as $S_t(T-t)$. While CDS contracts are written with a non-zero up-front fee and fixed coupons, the quoting convention is

[3] Duan, J.-C., J. Sun, and T. Wang (2012), Multiperiod Corporate Default Prediction — A Forward Intensity Approach. *Journal of Econometrics*, 170, 191-209.

[4] Duan, J.-C. and A. Fulop (2013), Multiperiod Corporate Default Prediction with the Partially-Conditioned Forward Intensity. National University of Singapore Working Paper.

[5] RMI-CRI Technical Report, 2013, Version 2013 Update 2b (http://d.rmicri.org/static/pdf/2013update2.pdf) National University of Singapore Risk Management Institute.

to still quote CDS in terms of the par spread. One can convert between the up-front fee convention and the par spread[6].

A CDS contract comprises the premium and protection legs (see Figure 3). The protection buyer typically pays premium payments to the seller on a quarterly basis, which are calculated based on the fixed spread until the reference entity defaults. In exchange, the protection buyer receives a contingent lump sum payment at the default time of the reference entity, and the settlement amount is based on the recovery rate on the reference instrument determined at the credit event auction held specifically for the defaulted obligor within one month of default.

CDS premium payments are subject to day count conventions similar to coupon bonds, but in fact accrual periods do not always end on payment dates. Differences are typically one day, and may be ignored for many practical purposes. For a more precise modeling of CDS, however, a set of accrual period end dates $\{t'_1, t'_2, \dots, t'_k\}$ and $A(t_{i-1}, t'_i)$ are introduced for the length of an accrual period measured as the fraction of a year using the appropriate day count convention.

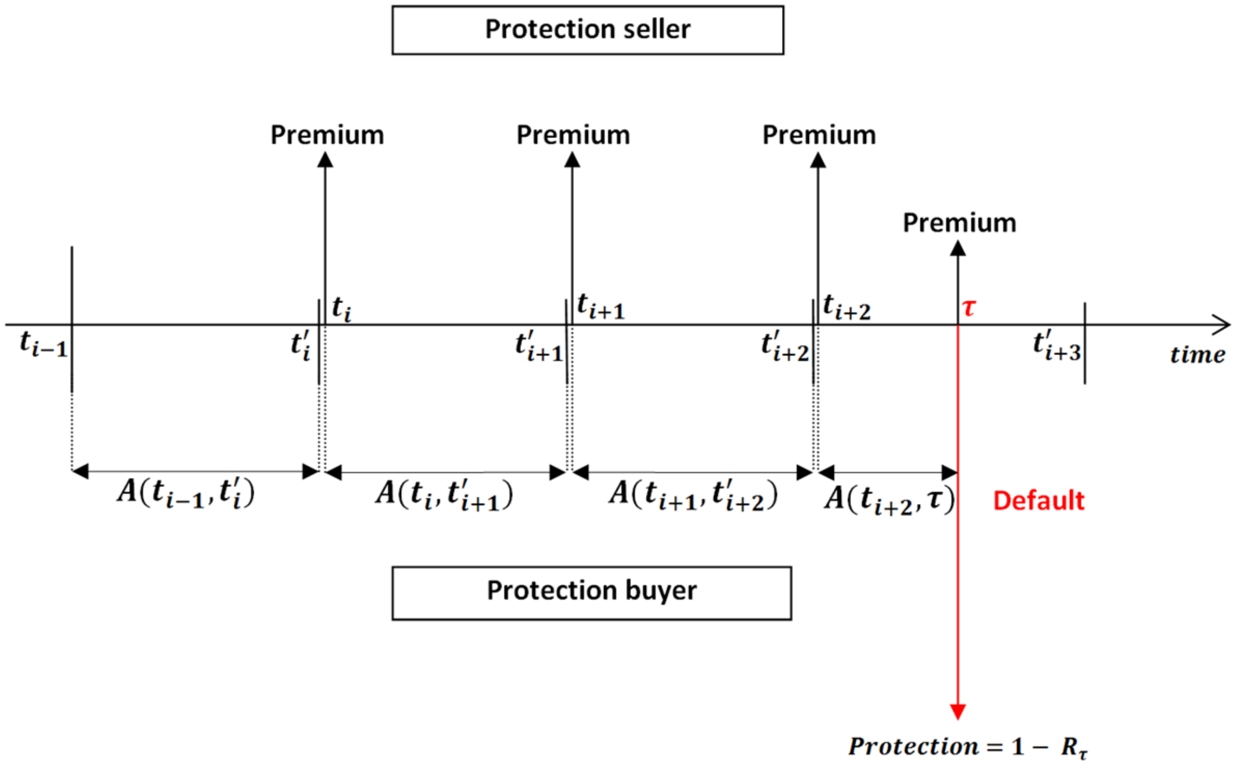

Figure 3: CDS protection cash flow for a sample contract with the default event at τ. The protection seller receives premium payments from the protection buyers. The protection buyer receives the amount $1 - R_\tau$ at default.

[6] Appropriate calculator can be found on http://www.cdsmodel.com/cdsmodel/fee-computations.html?.

ACTUARIAL PAR SPREAD

The formula for CDS par spread is presented in detail in the Appendix. Knowing the present value of the payment for the protection leg up to and including the final accrual end date, and the present value of the payments for the premium leg with the amount determined by the CDS par spread that is known when the contract is entered, we can obtain the CDS par spread $S_t(T-t)$. By replacing the risk-neutral probability measure Q with the physical probability measure P we can write the new actuarial par spread operator as $E_t^P(\cdot)$. This way one gets the actuarial par spread $S_t^{(a)}(T-t)$ by the assumption that all agents are truly risk-neutral and valuation are conducted purely on an actuarial basis.

OTHER-EXIT CASE

In order to compute the actuarial spread one needs to calculate expectations using the physical probability measure that also takes into account other-exit cases like mergers and acquisitions. The above mentioned papers by Duan, Sun and Wang (2012) and Duan and Fulop (2013) model other exits in terms of two independent Poisson intensities: f_t for default, and h_t for the other-exit, but in case of CDS, other-exit does not mean termination, because the successor entity will be assigned to replace the reference entity. The effect of the successor reference entity substituting for the predecessor reference entity in a CDS contract has been dealt with by Duan (2014) .

In the other-exit case, the CDS protection is typically shifted to the merged or acquiring entity, and the CDS will be triggered by the successor entity's subsequent default or other-exit. This probability of default or other-exit can be referred to as 'forward termination probability'. The succession entity of course cannot be determined beforehand. Therefore, some reasonable assumptions will be needed for implementation. One can, for example, assume the successor to be an entity that shares the same forward termination probability as the reference obligor, or an entity that is of median quality. In principle, succession may occur multiple times, which in turn requires us to make a further assumption on all subsequent substitutions. To make calculations possible, it is assumed that all subsequent successors must be of the same type in terms of the forward termination probability.

Under this assumption one can obtain more accurate values of actuarial spread for the case of other-exit.

The RMI-CRI actuarial spread is computed with the assumption that when the obligor exits for reasons other than default, the CDS protection is shifted to the merged or acquiring entity, with the successor potentially experiencing subsequent default or other exit with the same forward termination probability as the original obligor. Readers are referred to Duan (2014) for the solution of the actuarial spread under this assumption.

APPLICATION

The following section provides an example of how one can use the actuarial spread as an empirical pricing tool for CDS. The idea and the example are taken from Duan (2014), which used Eastman Kodak to demonstrate that there is an empirical relationship between market pricing of CDS and its actuarial spread. This relationship can be used for empirical pricing or for credit benchmarking - even of companies which have no CDS quotations or which have CDS that are not liquid enough to assess the risk of default in a timely manner.

EMPIRICAL PRICING OF CDS

Observing the market CDS spread, one would like to assess whether this instrument is under or overvalued, especially when the underlying entity approaches default and the market CDS price reflects the high level of risk. In this case, the CDS price is usually very high, as was demonstrated for example in the case of Greece. However, a high price does not necessary coincide with a high probability of default. In this case, a trader involved in the CDS market will be interested to know the appropriate price level. In the absence of liquidity in the market, this task will be even more challenging. One possible solution is to compare the market CDS spread with the actuarial spread. This method is further explained using the Eastman Kodak example to show how via the empirical relationship between these two spreads one can calculate whether the CDS is over or under priced by the market participants.

Eastman Kodak filed for Chapter 11 bankruptcy protection on January 19, 2012 and later emerged from bankruptcy on September 3, 2013. Its shares began to trade under a different ticker in NYSE on November 1, 2013. In Figure 4, Kodak's market and actuarial spreads are plotted for the last year before its bankruptcy filing.

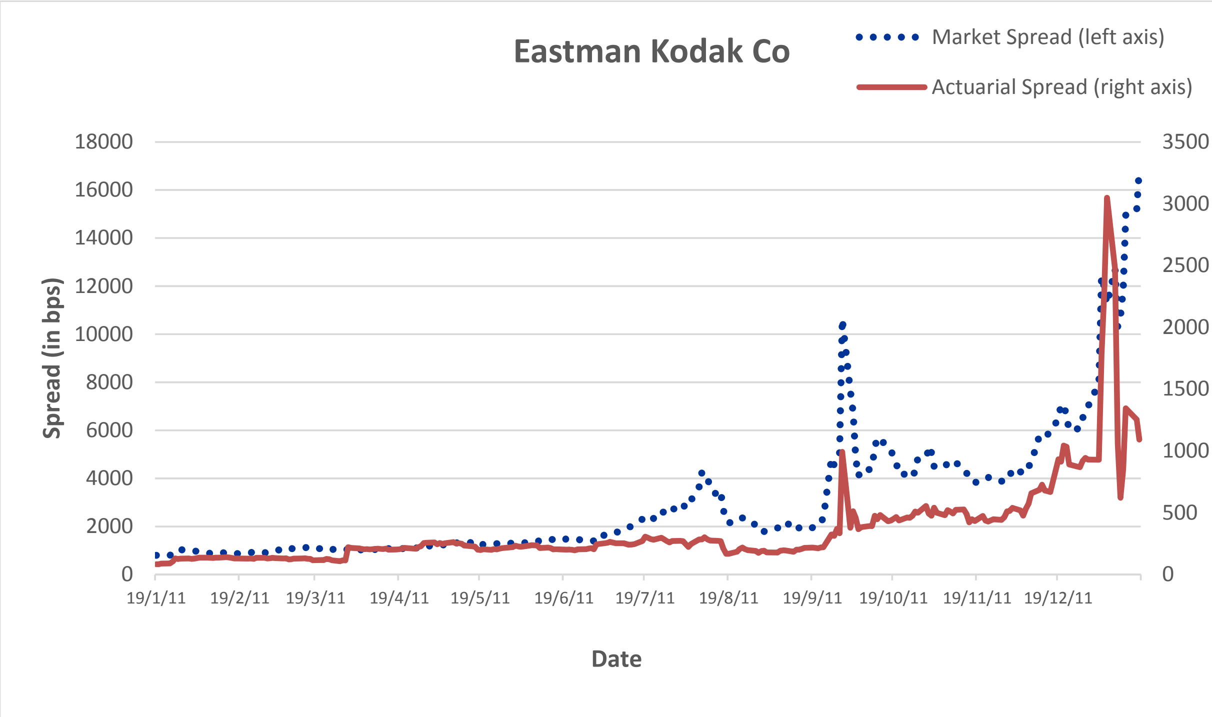

Figure 4: One-year daily time series of 5-year CDS market spread and actuarial spreads leading up Eastman Kodak's bankruptcy filing on January 19, 2012.

For example, its CDS was priced 4009.84 bps on November 16, 2011, and the actuarial spread is calculated to be 422.66 bps by assuming a recovery rate of 40%.

In Figure 5 the log-ratio of the CDS spread over its corresponding actuarial spread is plotted. As the figure shows, the log spread ratio hovers around its mean of 2.09. If we were to price the CDS on the trade date of November 16, 2011 using the actuarial par spread of 422.66 bps and the average log spread ratio of 2.09, we would obtain a predicted CDS spread of 3406 bps (i.e., $423 \times e^{2.09}$).

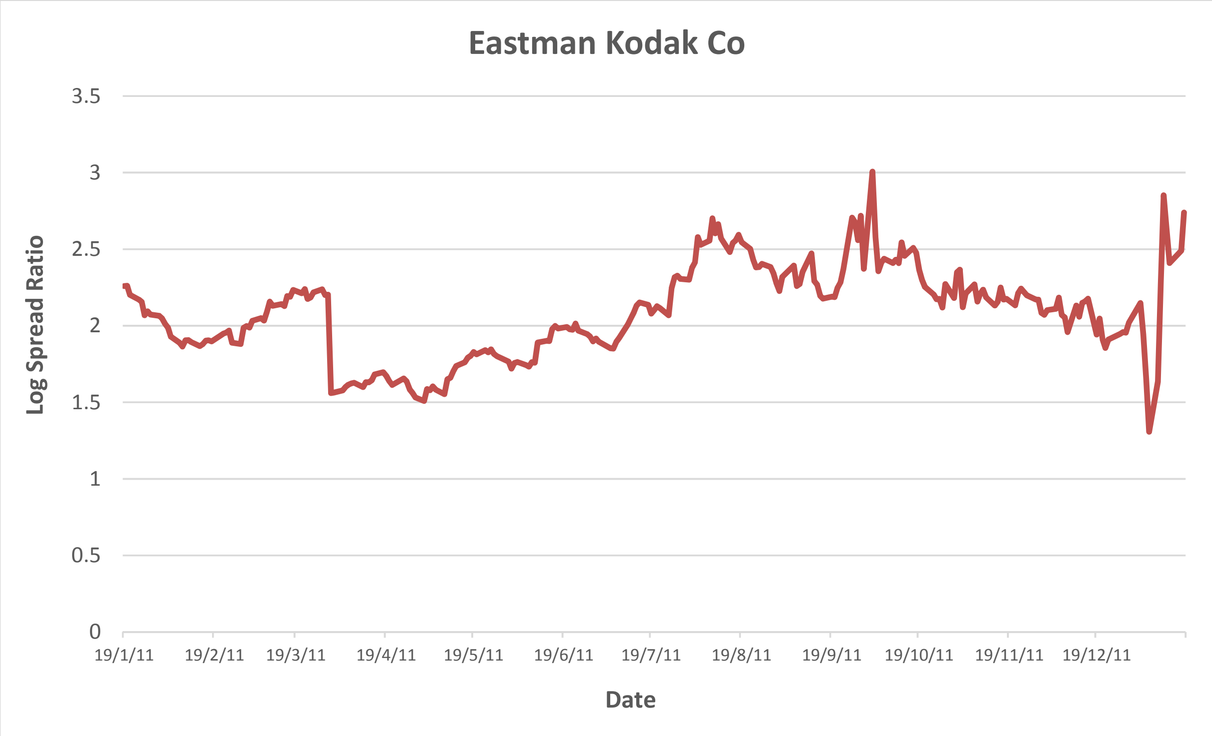

Figure 5: Time series of the log-ratio of the 5-year CDS spreads over their corresponding actuarial spreads leading up to Eastman Kodak's bankruptcy filing on January 19, 2012.

Alternatively we can use the simple lagged regression to obtain the log ratio of the CDS spread over its corresponding actuarial spread with the predictive equation:

$$ln\left(\frac{S_t}{S_t^{(a)}}\right) \approx 0.1487 + 0.9296 \times ln\left(\frac{S_{t-1}}{S_{t-1}^{(a)}}\right),$$

with $R^2 \approx 85\%$.

In Figure 6, the visual relationship is clearly reflective of the above lagged regression.

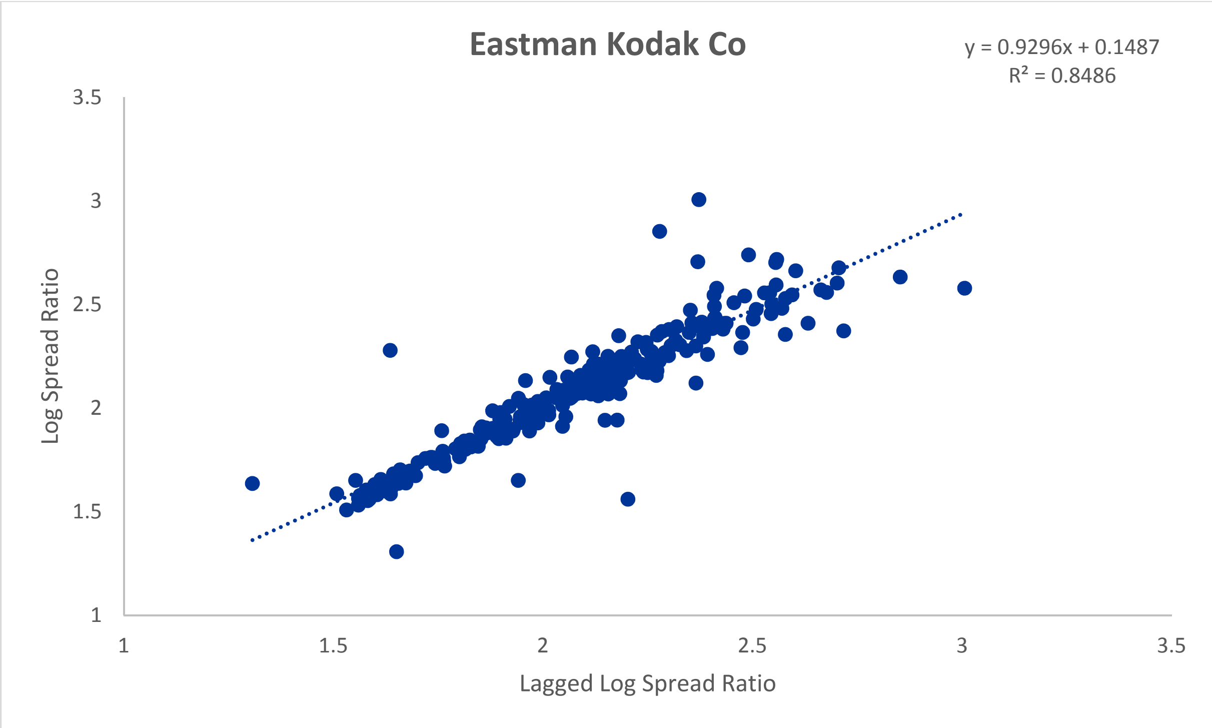

Figure 6: The log-ratio of the 5-year CDS spread over its corresponding actuarial spread versus the same log-ratio on the previous trading day.

Using the log spread ratio of 2.15 on November 15, 2011 (CDS spread = 4229 and actuarial spread = 490 bps) one obtains a predicted CDS spread of 3637 bps (i.e. $423 \times e^{2.15}$). As compared to the spread of 3406 bps that is directly predicted using the average log spread ratio of 2.09 and the actuarial spread of 423 bps, this alternative predicted spread is 231 bps closer to the observed CDS spread of 4001 on November 16, 2011.

From the above example it is clear that the log-ratio of the CDS spread over its corresponding actuarial par spread is highly predictable using its lagged value, and this predictive relationship can form a good basis for empirical pricing of CDS.

Alternatively, actuarial spreads can be used for risk benchmarking. This approach solely depends on the likelihood of default under the physical probability measure and thus is not complicated by risk premium and market liquidity. Also, the recovery rate can be easily adjusted to one's preference; for example, setting it to zero to focus on the probability of default by the obligor. An additional benefit of using the Actuarial Spread as a risk benchmark is that it is available for a much larger set of firms when comparing to the use of CDS as a risk benchmark.

APPENDIX

Denoting by $D_t(T-t)$ the appropriate money market discount factor starting from time t to some future date T one can adopt the familiar derivatives pricing theory with which there exists a risk-neutral pricing measure, and derivatives can be priced by taking expectation of its contingent payment with respect to the risk-neutral pricing measure as if economic agents were not risk adverse. Such a risk-neutral expectation operator at time t can be denoted by $E_t^Q(\cdot)$ to reflect the time-t information set.

In situations when no money has changed hands initially between the protection buyer and seller, the expected present value of the default leg up to and including the final accrual end date must equal the expected present value of the payment leg with the amount determined by the CDS par spread that is known when the contract is entered:

$$E_t^Q\left[(1-R_\tau)D_t(\tau-t)1_{\{t<\tau\leq t_k'\}}\right]$$
$$= S_t(T-t)\sum_{i=1}^{k}\left\{A(t_{i-1}\vee t, t_i')E_t^Q\left[D_t(t_i-t)1_{\{t_i'<\tau\}}\right] + E_t^Q\left[A(t_{i-1}\vee t,\tau)D_t(\tau-t)1_{\{t_{i-1}'<\tau\leq t_i'\}}\right]\right\},$$

where $t_{i-1}\vee t$ denotes the maximum of t_{i-1} and t so that the partial accrual for the first payment period from t_0 to t is taken out. For all pre-scheduled payments, t_i denotes the payment date, and t_i' is used for discounting and the accrual end date.

Solving for $S_t(T-t)$ one can write the CDS par spread as:

$$S_t(T-t) = \frac{E_t^Q\left[(1-R_\tau)D_t(\tau-t)1_{\{t<\tau\leq t_k'\}}\right]}{\sum_{i=1}^{k}\left\{A(t_{i-1}\vee t, t_i')E_t^Q\left[D_t(t_i-t)1_{\{t_i'<\tau\}}\right] + E_t^Q\left[A(t_{i-1}\vee t,\tau)D_t(\tau-t)1_{\{t_{i-1}'<\tau\leq t_i'\}}\right]\right\}}.$$

Denoting the time-t risk-free forward discount rate $r_t(s,q)$ starting at time $t+s$ with duration of $q-s$ where $q\geq s$. By implying the standard term structure theory $r_t(0,T-t) = -\frac{1}{T-t}\ln\left(E_t^Q[D_t(T-t)]\right)$. Assuming that R_τ,τ and $D_t(t_i-t)$ are all independent and let $\bar{R}_t = E_t^Q(R_\tau)$ one can write:

$$S_t(T-t) = \frac{(1-\bar{R}_t)E_t^Q\left[(1-R_\tau)D_t(\tau-t)1_{\{t<\tau\leq t_k'\}}\right]}{\sum_{i=1}^{k}\left\{A(t_{i-1}\vee t, t_i')E_t^Q\left[D_t(t_i-t)1_{\{t_i'<\tau\}}\right] + E_t^Q\left[A(t_{i-1}\vee t,\tau)D_t(\tau-t)1_{\{t_{i-1}'<\tau\leq t_i'\}}\right]\right\}}.$$

For the actuarial spread, denoted by $S_t^{(a)}(T-t)$ we have:

$$S_t^{(a)}(T-t) = \frac{(1-\bar{R}_t)E_t^P\left[(1-R_\tau)D_t(\tau-t)1_{\{t<\tau\leq t_k'\}}\right]}{\sum_{i=1}^{k}\left\{A(t_{i-1}\vee t, t_i')E_t^P\left[D_t(t_i-t)1_{\{t_i'<\tau\}}\right] + E_t^P\left[A(t_{i-1}\vee t,\tau)D_t(\tau-t)1_{\{t_{i-1}'<\tau\leq t_i'\}}\right]\right\}},$$

which is the par spread computed by replacing the risk-neutral probability measure Q with the physical probability measure P.