Indexing Executive Stock Options Relatively *

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Abstract

We propose and analyze a relative indexing scheme for executive stock options. In contrast to the absolute indexing scheme of Johnson and Tian (2000) which bases the option's payoff on the dollar difference between the stock price and the indexed exercise price, our scheme defines the option's payoff on the ratio of the stock price over the indexed exercise price. The absolute indexing is shown to still yield an option value moving in tandem with the index, which is undesirable for it ends up rewarding or penalizing the executive for the factors outside his/her control. Besides correcting the misalignment between reward and performance, relative indexing also possesses many desirable incentive properties. On a per dollar basis, it has a much higher delta (i.e., sensitivity to stock price) and vega (i.e., sensitivity to firm volatility) than absolute indexing, implying that relative indexing has a stronger performance incentive and is more effective in encouraging the CEO to undertake more risky projects which otherwise may be shun due to his/her less-diversified personal wealth.

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1. Introduction

Executive stock options have received unprecedented attention in the recent years from academic researchers, policy makers, boards of directors, and the investment public in general. One of the issues being debated is whether / how to design compensation schemes based on relative performance. Although the vast majority of stock options have fixed exercise prices and hence reward absolute performance, attention has been drawn to stock options whose exercise prices are linked to certain benchmark indices. Rappaport (1999) discussed the general idea and related issues of indexing. Johnson and Tian (2000) designed and analyzed an indexing scheme for stock options.

The logic behind indexing the exercise price is both appealing and simple. To illustrate, suppose an at-the-money call option is granted to a CEO with an exercise price of $100. At maturity, the CEO would pocket $20 if the stock price has gone up by 20%. Yet over the same period, the overall stock market return may be, say, 40%. The firm's stock has under-performed the market, and the $20 compensation to the CEO is thus not justified. Likewise, if the firm's stock and the market have declined by 10% and 30% respectively, the CEO would not have received any compensation since the option expires out-of-the-money, yet the CEO has outperformed the market. Had the exercise price been indexed to the market, the above undue compensation or penalty would have been avoided. Indexing according to Johnson and Tian (2000) would give rise to an exercise price of $140 in the first case if the firm's stock has a beta of one, which in turn renders the option out-of-the-money. In the second case, the indexed exercise price would become $70 and lead to a option payoff of $20. Such indexing appears to have offered a compensation commensurate with the firm's relative performance.

There is a serious problem with such an indexing scheme, however. Meulbroek (2001) pointed out that the option value is homogeneous of degree one with respect to the stock price and the exercise price. The option's value would increase even when the stock price and the indexed exercise price increase by the same factor. For instance, suppose an option is granted 20% in-the-money with the current stock price and the exercise price being $120 and $100 respectively and the stock has a beta of one. If neither the stock nor the market index has changed (relative to their initial levels) by the option's maturity, then the payoff is \$20, the same as the intrinsic value at the granting time. If the stock and the market have both increased by 10%, the option's payoff should still be \$20 since the stock did not outperform the market. However, the option's payoff is actually \$120(1.1) - \$100(1.1) = \$22, which has also increased by 10%. It is easy to see that a similar effect exists when the stock and the market decline by the same percentage point. In fact, such effect exists at any time before the option's maturity: holding other things constant, the option's fair value would increase or decrease in tandem with the simultaneous movements in the stock price and the exercise price. In short, such an indexing scheme does not achieve its intended purpose.

A moment of reflection reveals the root of the problem: the indexing scheme proposed by Rappaport (1999) and Johnson and Tian (2000) adjusts the *absolute* level of the exercise price, but it is the *relative* level that truly captures the relative performance. Hereinafter, we will refer to their scheme as *absolute indexing*, and the scheme proposed in this paper *relative indexing*. Absolute indexing focuses on the dollar value of the exercise price, and therefore can't determine whether and by how much the stock has under-performed or outperformed the market. As a result, the CEO may be rewarded for poor relative performance and penalized for good relative performance.

There are two key aspects in indexing - market volatility effect and market level effect. What absolute indexing actually achieves is the removal of the market volatility from compensation. As a result, the option contract is effectively linked to the volatility of the stock relative to the market. Simply put, absolute indexing has succeeded in handling the volatility effect but failed to address the market level effect.

We propose a relative indexing scheme which is a simple modification of the Johnson and Tian (2000) indexing approach. Instead of focusing on the absolute level of the stock price and the exercise price, we take as the underlying variable the ratio of the stock price over the index level, i.e., treating the market index as a numeraire asset. The option contract is then structured around this variable intending to measure the relative performance of the stock in relation to the market. Relative indexing is shown to simultaneously deal with both the market volatility and level effects, in the sense that the incentive contract no longer rewards or punishes the executive for an outcome driven by the overall market.

We find that absolute indexing either rewards too generously or penalizes too harshly. The only situation in which it presents a fair compensation is when the stock price and the index never change during the life of the option, the least interesting situation. In addition, the misalignment between performance and compensation is bigger when the firm's beta or systematic risk is high. We show that relative indexing can correct the above deficiencies. Relative indexing is also more desirable than absolute indexing in terms of incentive effects. To begin with, on a per dollar basis, relative indexing's delta (i.e., the option's sensitivity to the stock price) is much higher than its absolute indexing counterpart, reflecting a stronger performance incentive. Relative indexing also leads to a higher vega (i.e., the option's sensitivity to firm's volatility), which means it is more effective than absolute indexing in inducing CEO's to undertake risky projects. This is useful since CEO's tend to shun projects that are risky but beneficial to the firm, due to their less-diversified personal wealth. Finally, relatively indexed option is also superior to its absolutely indexed counterpart in preserving its value against the passage of time, i.e., it has a lower rate of time decay. Our theoretical findings are backed by a case study based on real market data.

The rest of the paper is organized as follows. Section 2 briefly reviews the absolute indexing scheme of Johnson and Tian (2000), and then presents the relative indexing scheme. A comparison of the two schemes is also given in this section. Section 3 examines the incentive effects of the two indexing schemes. Section 4 presents a case study based on real market data. Section 5 concludes the paper.

2. Executive Stock Options: Relative versus Absolute Indexing

2.1. Absolute Indexing — A Brief Review

In this section, we review the absolute indexing scheme of Johnson and Tian (2000). Suppose there exists an index which can capture the systematic risk of the firm's stock. The stock price S_t and the index price I_t are assumed to follow a joint geometric Brownian motion:

$$\frac{dS_t}{S_t} = (\mu_S - q_S)dt + \sigma_S dz_{S,t}, \tag{2.1}$$

and

$$\frac{dI_t}{I_t} = (\mu_I - q_I)dt + \sigma_I dz_{I,t}, \tag{2.2}$$

with

$$dz_{S,t}dz_{I,t} = \rho dt,$$

where μ_S, q_S and σ_S are respectively the expected return, dividend yield and volatility of the firm's stock, μ_I, q_I and σ_I are the index counterparts, and $z_{S,t}$ and $z_{I,t}$ are standard Wiener processes with correlation ρ. Furthermore, we define $\beta \equiv \rho\frac{\sigma_S}{\sigma_I}$.

Johnson and Tian (2000) defined the indexed exercise price at time t as

$$H_t = S_0 \left(\frac{I_t}{I_0}\right)^{\beta} e^{\eta t}, \tag{2.3}$$

where $\eta = (r - q_S) - \beta(r - q_I) + \frac{1}{2}\rho\sigma_S\sigma_I(1 - \beta)$, and r is the risk-free interest rate. The indexed exercise price is the expected stock price (with the expectation taken at the initial granting time) conditional on the realized index level and zero expected excess return. In other words, this specific choice of indexing is motivated by $E_0[S_t \mid I_t] = H_t$ when $\alpha \equiv \mu_S - r - \beta(\mu_I - r) = 0$, i.e., the stock is not expected to yield an abnormal return.

The payoff of a European style, indexed option at maturity is

$$\max[S_T - \lambda H_T, 0].$$

Parameter λ determines the moneyness of the option at the initial granting time. Since $S_0 = H_0$, $\lambda = 1$ corresponds to the option being granted at-the-money, $\lambda > 1$ out-of-the-money, and $\lambda < 1$ in-the-money. By treating the indexed option as an exchange option and applying the formula derived by Margrabe (1978), the value of the indexed option at time t can be written as

$$C_t^{(A)} = e^{-q_S(T-t)} \left[S_t N(d_{1,t}) - \lambda H_t N(d_{2,t})\right], \tag{2.4}$$

where $N(\bullet)$ is the cumulative distribution function of a standard normal random variable, and

$$d_{1,t} = \frac{\ln(\frac{S_t}{\lambda H_t}) + 0.5\sigma_a^2(T-t)}{\sigma_a\sqrt{T-t}},$$
$$d_{2,t} = d_{1,t} - \sigma_a\sqrt{T-t},$$
$$\sigma_a = \sigma_S\sqrt{1-\rho^2}.$$

The option value in (2.4) is homogeneous of degree one with respect to the stock price S_t and the indexed exercise price H_t. Moreover, the critical volatility to the option value, i.e., σ_a, measures the stock price swing relative to the indexing numeraire H_t. In other words, σ_a is effectively a relative volatility. Holding other things constant, when S_t and H_t change by a factor x, the option value will also change by the same factor, an undesirable feature as far as rewarding the executive for his/her relative performance is concerned.

2.2. Relative Indexing — An Alternative Design

Absolute indexing only focuses on the level of the exercise price, and the option's payoff depends on the dollar difference between the stock price and the exercise price. In order to reward genuine superior performance, we need to link the option's payoff to the relative change between the stock price and the benchmark level. To this end, we propose the following payoff structure:

$$H_0 e^{(r-q_S)T} \max[\frac{S_T}{H_T} - \lambda, 0],$$

where $H_0 = S_0$ and $H_0 e^{(r-q_S)T}$ determines the dollar size of the contract. By design, the relative performance variable $\frac{S_T}{H_T}$ has removed the net growth factor in the stock and the benchmark which is the risk-free interest rate minus the stock's dividend yield. It is therefore natural to include $e^{(r-q_S)T}$ in the dollar size definition to compensate for the discounting effect due to making the contingent payment at the future date, T. We refer to this indexing scheme as relative indexing. It is straightforward to show that the value of this option at time t is

$$C_t^{(R)} = H_0 e^{rt-q_S T}\left[\frac{S_t}{H_t}N(d_{1,t}) - \lambda N(d_{2,t})\right], \quad (2.5)$$

where $d_{1,t}$ and $d_{2,t}$ have been defined earlier. The relationship between the option values under the two indexing schemes is

$$C_t^{(R)} = \frac{H_0 e^{(r-q_S)t}}{H_t} C_t^{(A)}. \tag{2.6}$$

It is apparent from (2.6) that, the two options have equal values at the initial granting time. Afterwards, apart from the deterministic component, the option value under relative indexing equals that under absolute indexing scaled by the indexing benchmark H_t. This feature is attractive because it removes the market level effect so that the option value becomes homogeneous of degree zero in I_t, i.e., the option value is unaffected by the movement of the market. This assertion will be formally established in the following subsection.

2.3. Comparisons Between Absolute Indexing, Relative Indexing and No-indexing

Using (2.1) and (2.2), we have the following relationship:

$$\ln\frac{S_t}{S_0} = \alpha t + r(1-\beta)t - (q_S - \beta q_I)\, t - \frac{1}{2}\left(\sigma_S^2 - \beta\sigma_I^2\right) t + \beta \ln\frac{I_t}{I_0} + \varepsilon_t \tag{2.7}$$

where ε_t has mean 0 and standard deviation $\sigma_a\sqrt{t}$. Recall that $\alpha \equiv \mu_S - r - \beta\,(\mu_I - r)$, defining the stock's abnormal performance. It is well known that in the equilibrium of a single-index economy, α should equal 0, i.e., there should be no abnormal performance. In general, however, abnormal performance may exist, i.e., α need not be zero. Thus,

$$\begin{aligned} \ln\frac{S_t}{H_t} &= \ln\frac{S_t}{S_0} - \beta\ln\frac{I_t}{I_0} - \eta t \\ &= \left[\alpha - \frac{1}{2}\sigma_S^2(1-\rho^2)\right] t + \varepsilon_t. \end{aligned} \tag{2.8}$$

The above expression and (2.5) imply that the value of the relatively indexed option, $C_t^{(R)}$ is independent of the market index level I_t because it is a function of $\frac{S_t}{H_t}$ which is in turn a function of the firm specific risk ε_t. Since $C_t^{(R)}$ is increasing in $\frac{S_t}{H_t}$ which is in turn increasing in α, the relatively indexed option rewards or penalizes abnormal performance depending on $\alpha > 0$ or $\alpha < 0$.

Since $d_{1,t}$ and $d_{2,t}$ take the same form under both indexing schemes, the effect of removing market volatility is identical for both schemes. The difference though is on the market level effect. In the case of absolute indexing, $C_t^{(A)}$ is an increasing function of I_t. In other words, the executive is rewarded or penalized for something beyond his/her control. In contrast, relative indexing successfully deals with the market level effect; that is, $C_t^{(R)}$ does not respond to the movement of I_t, and thus no undue compensation or penalty is imposed for factors beyond the executive's control.

To illustrate the above point, we simulate the bivariate system in (2.1) and (2.2) to obtain $5,000$ pairs of I_t and S_t and use them to compute the corresponding option values at time t. We then plot the option values against the realized index and stock prices. The simulation parameters are: $S_0 = I_0 = \$100$, $t = 0.05$, $T - t = 9.95$, $\sigma_S = 0.20$, $\sigma_I = 0.15$, $\rho = 0.75$, $r = 0.08$, $q_S = q_I = 0.02$, $\mu_I = 0.12$, $\lambda = 1.0$.[1] The stock's expected return is calculated via $\mu_S = r + \beta(\mu_I - r)$ by setting $\alpha = 0$. We choose a smaller t so that the plots are not overwhelmed by the randomness in I_t and S_t. Figure 1 presents the plots. Panel A plots the option values against the stock prices. The plot reveals that higher stock prices are generally associated with higher option values, as should be. It is also apparent that the slope for the absolutely indexed option is more pronounced than its relatively indexed counterpart. Moving to Panel B which plots the option values against the index prices, it is clear that the relatively indexed option does not depend on the index level. Deviations from the horizontal line are simply due to the idiosyncratic risk. In contrast, we see a positive association between option values and the index realizations for the absolutely indexed option. The above observations illustrate the drawback of absolute indexing: the option contract awards or penalizes the executive for market-wide factors. Notice that we have produced plots at different correlations or betas and different abnormal performances (i.e., $\alpha \neq 0$). Other than upward or downward location changes, the plots maintain the same profile. In summary, Figure 1 clearly demonstrates how relative indexing can correct the undesirable level effect inherent in absolute indexing.

To further demonstrate the key point of the paper, we report in Table 1 how indexed option values respond to instantaneous changes in the stock price and the index. Specifically, we assume that there are no changes up to time t, but the stock price and the exercise price undergo instan-

[1] Unless otherwise specified, we use the same parameter values for subsequent numerical analyses.

taneous changes at time t. This design of the numerical exercise will avoid the complication of time decay in option values. Table 1 reports results for options which were granted five years ago and have another five years to go before maturity.[2] We also examined other combinations and the quantitative conclusions are the same.

The most telling part of the table is the middle panel with $\beta = 1$. Here, the benchmark and the index change by the same percentage point. As a result, the stock price and the benchmark or the exercise price will change by the same percentage point as long as the index also change by the same amount. It is seen that when the stock and the index change by the same percentage point (increase or decrease), the option value under absolute indexing would change by the same amount. In contrast, the option with relative indexing does not experience any change in this case. This means that relative indexing can accomplish the intended purpose of relative performance compensation while absolute indexing cannot. It is clear from the table that, with absolute indexing, the larger the changes in the stock price and the index, the bigger the unjustified change in the option value. When the stock price goes up chiefly due to a bullish market, the CEO is rewarded although he/she did not contribute to the stock's performance. When the firm's stock loses value simply due to a bearish market, the CEO is penalized for his/her normal or perhaps better performance.

More generally, we can see the deficiencies of absolute indexing through other combinations of changes. When the stock price increases by less than the index does (e.g., 8% vs. 10%), the option value under absolute indexing even increases slightly, but the option value under relative indexing goes down, as a well-designed incentive option should be. When the stock price increases by more than the index does (e.g., 10% vs. 8%), the option value under absolute indexing increases much more than the option value under relative indexing does. But this magnitude of increase is not warranted. On the other hand, when the stock price decreases by less than the index does (e.g., -8% vs. -10%), the option value under absolute indexing even decreases slightly, but the option value under relative indexing goes up. This outcome is desirable because the firm has outperformed the market. When the stock price decreases by more than the index does (e.g., -10% vs -8%),

[2] Following Johnson and Tian (2000), we vary firm volatility σ_S to obtain different beta's.

absolute indexing would bring the option value down much more than the relative indexing does, which penalizes the CEO unfairly. In summary, absolute indexing either rewards too generously or penalizes too harshly. The only situation in which it represents a fair compensation is when the stock and the market stay put (i.e., 0% change in both), a situation which is unlikely to occur and is of no interest to any compensation design.

When the stock's beta is not unity, absolute indexing exhibits similar biases. It is seen that, for the same percentage change in the index, a larger beta leads to a bigger percentage change in the benchmark. Meanwhile, a bigger divergence between the changes in the stock price and the benchmark corresponds to a bigger valuation bias by absolute indexing. As a result, the higher the beta, the bigger the bias associated with absolute indexing. We can therefore infer that firms having a higher systematic risk will suffer from the worst misalignment between compensation and performance if they use absolute indexing for stock options. For instance, when the stock and the index decline by 8% and 10% respectively, the CEO's option package would suffer a 3.11% loss under relative indexing if the stock's beta is 0.75. If the firm uses absolute indexing, the loss would be 10.47%, in which case the CEO suffers an additional 7.36% (=10.47% − 3.11%) loss over and above the fair loss. This number becomes 14.58% (=18.15% − 3.57%) if the stock's beta is 1.25.

Lastly, as demonstrated by Johnson and Tian (2000), the correlation is a key parameter determining the value of an indexed option. For both types of indexed options, at the time of granting, maximum value is achieved when the correlation between the stock and the index is zero. Once the options are granted, a higher correlation will lead to a lower value for both types of options due to the lower effective volatility $\sigma_a = \sigma_S\sqrt{1-\rho^2}$. We omit the plots for brevity.

3. Incentive Effects

In addition to the valuation effects discussed in the preceding section, relative indexing generates different incentive effects. As shown by Johnson and Tian (2000), under absolute indexing, the comparative statics are

$$
\begin{aligned}
\frac{\partial C_t^{(A)}}{\partial S_t} &= e^{-q_S(T-t)}N(d_{1,t}),\\
\frac{\partial C_t^{(A)}}{\partial I_t} &= -\lambda\beta\frac{S_0}{I_0}\left(\frac{I_t}{I_0}\right)^{\beta-1}e^{\eta t-q_S(T-t)}N(d_{2,t}),\\
\frac{\partial C_t^{(A)}}{\partial \sigma_S} &= \lambda H_t e^{-q_S(T-t)}\left(\sqrt{(1-\rho^2)(T-t)}N'(d_{2,t})-\left[\frac{\rho}{\sigma_I}\ln\left(\frac{I_t}{I_0}\right)+t\left(\frac{\rho\sigma_I}{2}-\rho^2\sigma_S-(r-q_I)\frac{\rho}{\sigma_I}\right)\right]N(d_{2,t})\right),\\
\frac{\partial C_t^{(A)}}{\partial \sigma_I} &= \lambda H_t e^{-q_S(T-t)}\left[\frac{\rho\sigma_S}{\sigma_I^2}\ln\left(\frac{I_t}{I_0}\right)-t\left(\frac{\rho\sigma_S}{2}+(r-q_I)\frac{\rho\sigma_S}{\sigma_I^2}\right)\right]N(d_{2,t}),\\
\frac{\partial C_t^{(A)}}{\partial \rho} &= -\lambda H_t e^{-q_S(T-t)}\left(\frac{\rho\sigma_S\sqrt{T-t}}{\sqrt{1-\rho^2}}N'(d_{2,t})+\left[\frac{\sigma_S}{\sigma_I}\ln\left(\frac{I_t}{I_0}\right)+t\left(\frac{\sigma_S\sigma_I}{2}-\rho^2\sigma_S-(r-q_I)\frac{\sigma_S}{\sigma_I}\right)\right]N(d_{2,t})\right),\\
\frac{\partial C_t^{(A)}}{\partial t} &= q_S C_t^{(A)}-\frac{\sigma_a e^{-q_S(T-t)}}{2\sqrt{T-t}}\lambda H_t N'(d_{2,t})-\lambda\eta H_t e^{-q_S(T-t)}N(d_{2,t}),\\
\frac{\partial C_t^{(A)}}{\partial r} &= t(\beta-1)\lambda H_t e^{-q_S(T-t)}N(d_{2,t}),
\end{aligned}
$$

where $N'(\cdot)$ stands for the standard normal density function.[3]

By equation (2.6), we have the following corresponding comparative statics under relative indexing:

$$
\begin{aligned}
\frac{\partial C_t^{(R)}}{\partial S_t} &= \frac{H_0 e^{(r-q_S)t}}{H_t}\frac{\partial C_t^{(A)}}{\partial S_t},\\
\frac{\partial C_t^{(R)}}{\partial I_t} &= \frac{H_0 e^{(r-q_S)t}}{H_t}\frac{\partial C_t^{(A)}}{\partial I_t}-\frac{\beta}{I_t}C_t^{(R)},\\
\frac{\partial C_t^{(R)}}{\partial \sigma_S} &= \frac{H_0 e^{(r-q_S)t}}{H_t}\frac{\partial C_t^{(A)}}{\partial \sigma_S}-C_t^{(R)}\left[\frac{\rho}{\sigma_I}\ln\left(\frac{I_t}{I_0}\right)+t\left(\frac{\rho\sigma_I}{2}-\rho^2\sigma_S-(r-q_I)\frac{\rho}{\sigma_I}\right)\right],\\
\frac{\partial C_t^{(R)}}{\partial \sigma_I} &= \frac{H_0 e^{(r-q_S)t}}{H_t}\frac{\partial C_t^{(A)}}{\partial \sigma_I}-C_t^{(R)}\left[-\frac{\rho\sigma_S}{\sigma_I^2}\ln\left(\frac{I_t}{I_0}\right)+t\left(\frac{\rho\sigma_S}{2}+(r-q_I)\frac{\rho\sigma_S}{\sigma_I^2}\right)\right],\\
\frac{\partial C_t^{(R)}}{\partial \rho} &= \frac{H_0 e^{(r-q_S)t}}{H_t}\frac{\partial C_t^{(A)}}{\partial \rho}-C_t^{(R)}\left[\frac{\sigma_S}{\sigma_I}\ln\left(\frac{I_t}{I_0}\right)+t\left(\frac{\sigma_S\sigma_I}{2}-\rho^2\sigma_S-(r-q_I)\frac{\sigma_S}{\sigma_I}\right)\right],\\
\frac{\partial C_t^{(R)}}{\partial t} &= \frac{H_0 e^{(r-q_S)t}}{H_t}\frac{\partial C_t^{(A)}}{\partial t}+C_t^{(R)}\left[\beta(r-q_I)-\frac{1}{2}\rho\sigma_S\sigma_I(1-\beta)\right],\\
\frac{\partial C_t^{(R)}}{\partial r} &= \frac{H_0 e^{(r-q_S)t}}{H_t}\frac{\partial C_t^{(A)}}{\partial r}+t\beta C_t^{(R)}.
\end{aligned}
$$

Some of the partial derivatives are easy to sign while others are not. Instead of focusing on the sign, we will contrast numerically the comparative statics across the three types of options: absolute indexing, relative indexing and no-indexing. Following Johnson and Tian (2000), we adjust the number of indexed options so that all options have the same value at the granting time. We then multiply the indexed option's partial derivatives by the value adjustment factors.

Figure 2 presents the plots for "stock delta", the option value's sensitivity to the stock price. Here, the current index level is assumed to be \$100, and the option is five years old and has a remaining time to maturity of five years. When the indexed options are initially granted, the

[3] We have added $\frac{\partial C_t^{(A)}}{\partial I_t}$ which is not in Johnson and Tian (2000). In addition, we report the option's sensitivity to time by keeping the maturity time constant whereas Johnson and Tian (2000) reported the option's sensitivity to maturity by keeping the currrent time constant.

deltas of the two types of options are equal. However, as shown in Figure 2, once the options are granted, the relatively indexed option has a higher delta for all moneyness situations, meaning that this compensation contract is more responsive to the stock price movement when other factors are fixed. The more the options are in-the-money, the bigger the difference in delta's. Although not shown, figures with other time combinations (i.e., varying t, while keeping T equal to 10 years) reveal similar patterns. The observations lead to an important implication: relative indexing not only reflects the true spirit of compensation based on relative performance, but also brings about stronger incentives than absolute indexing. The latter is especially true for deep in-the-money options.

For completeness, we present in Figure 3 the plots for "index delta", the option value's sensitivity to the index price. As expected, index delta is negative for both types of indexed options. In addition, the relatively indexed option is much more sensitive to the index, especially when the option is deep-in-the-money. Figure 3 is essentially an alternative presentation of Figure 2, since the index constitutes the exercise price.

We have two vega's to present: the option's sensitivity to the firm volatility and that to the index volatility. We will discuss the former first. Figure 4 contains two plots, with Panel A for newly granted options ($t = 0$, $T - t = 10$), and Panel B for previously granted options ($t = 5$, $T - t = 5$). It is seen in Panel A that the two indexing schemes lead to the same vega initially, and at-the-money options have the highest vega. However, as shown in Panel B, the two schemes have drastically different vega's after the options are granted. When the options are out-of-the-money, vega increases with moneyness; when the options are in-the-money, vega still increases with moneyness under relative indexing, but remains unchanged under absolute indexing. Moreover, at all levels of moneyness, vega is larger for relative indexing. We can draw two implications from Figure 4. First, the risk incentive of relative indexing is higher than that of absolute indexing, both of which are higher than the risk incentive of no indexing. This means that relative indexing is the most effective in terms of inducing the CEO to take risky projects to enhance the firm value. (As discussed by Johnson and Tian (2000), CEO's tend to avoid risk taking due to their less diversified personal wealth.) Second, for previously granted options, risk incentive of relative indexing is higher when the option is in-the-money, in contrast with that of absolute indexing. This again is a desirable feature of relative indexing since CEO's, in order to protect their wealth, will have a tendency to avoid risky projects once the options are deep in-the-money (i.e., to protect one bird in hand instead of chasing two birds in the bush). The higher vega would induce adequate risk taking on the part of the CEO when the firm is successful.

As for the index volatility, as long as the option is not too new (i.e., t is not too small) and the index has not increased substantially (i.e., I_t is not much higher than I_0), vega is negative under both indexing schemes. This is easy to understand since σ_I only affects the benchmark level H_t, and for most situations, H_t is positively related to σ_I. Figure 5 is an index volatility counterpart of Figure 4. It is seen that the sensitivity to index volatility or the index vega becomes smaller (i.e., less negative) as the options become deep out-of-the-money. In addition, the index vega is larger (i.e., more negative) for relative indexing. Although the index volatility is not under the CEO's control, Figure 5 reveals that an option with relative indexing is more sensitive to the index volatility than its absolute indexing counterpart.

Next, we plot in Figure 6 the option's sensitivity to correlation against the stock prices. The plots are for two levels of correlation: $\rho = 0.5$ and $\rho = 0.9$, corresponding to a beta of 0.67 and 1.2 respectively. At time $t = 0$, the plots are similar to Figure 4 of Johnson and Tian (2000), and are omitted for brevity. We plot the sensitivity in Figure 6 for $t = 5$ and $T - t = 5$. It is seen that the correlation sensitivity is generally positive for in-the-money options, and is higher when the correlation itself is high. Moreover, as the option becomes deep in-the-money, the sensitivity levels off for absolute indexing but continues to increase for relative indexing. The above observations generally hold true for other combinations of t and $T-t$ (as long as t is not too small) and other levels of I_t.[4] There are two important implications. First, in contrast to what Johnson and Tian (2000) have concluded (viz, the CEO has an incentive to reduce the correlation to zero), the CEO will have an incentive to increase the correlation (or equivalently, the systematic risk) when the option

[4]Specifically, when t is small or when I_t is high, the general patterns in Figure 6 still obtain, except that we see a negative sensitivity in a wider range of moneyness, and the sensitivity becomes positive at a higher stock price.

is several years old and is deep in-the-money. This incentive is much higher with relative indexing. Johnson and Tian's conclusion only applies when $t = 0$, whereas our conclusion is applicable most of the time within the life of the option. Second, unlike absolute indexing, relative indexing is associated with a higher correlation sensitivity when the option becomes deep in-the-money.[5] This means that the CEO should be under a more watchful eye when the stock options are several years old and are deep in-the-money. The incentive to pursue a higher correlation means a higher systematic risk of the stock (holding other things equal), something the firm may not desire.

We now examine "theta", the option's sensitivity to the passage of time. Panel A of Figure 7 plots the sensitivity against the stock price by assuming $t = 5$, $T - t = 5$, while Panel B plots the sensitivity against time to maturity by assuming the same current time $t = 1$. In Panel B, the adjustment factors for indexed options are calculated by equating option values initially for each maturity (i.e., $t = 1$, $T - t = 1, 2, 3, 4, ...$). These factors are then applied to theta's of the indexed options.[6] As seen in Panel A, theta is negative for non-indexed options, reflecting the usual time decay effect. However, for deep in-the-money indexed options, the passage of time will actually increase the option value, and this effect is much more pronounced for relatively indexed options. Intuitively, this is similar to the time decay of a deep in-the-money European put option. When an indexed option is deep in-the-money, waiting can be costly because the index may go up, leading to a higher exercise price.

As seen in Panel B of Figure 7, the plots for the non-indexed option and the absolutely indexed option are basically mirror image of those in Johnson and Tian (2000) who examined the option's sensitivity to time to maturity. The plot for the relatively indexed option exhibits some interesting features. To begin with, the time decay is smaller than the other two options. Moreover, when

[5] A closer examination of the comparaitive statics for the correlation reveals the intuition behind the different patterns. To start with, $\frac{\partial H_t}{\partial \rho}$ is generally negative, which means a higher correlation should be associated with a higher option value due to a lower exercise price. However, a higher correlation also means a lower overall volatility in the pricing formula, which leads to a lower option value. When the option is deep in-the-money, the volatility impact diminishes, and the exerice price impact remains. This is why the correlation sensitivity levels off for absolute indexing. With relative indexing, there is also a "division effect", i.e., H_t is a divisor of the option value with absolute indexing. This effect makes the sensitivity increases with the moneyness.

[6] We also generated plots by assuming different values of t. Since they all share the same pattern, we omit them for brevity.

the maturity is long enough (longer than 6 years in this case), the option's value actually increases with the passage of time.[7] These observations lead to an important practical implication: in terms of preserving the option value against the passage of time, relatively indexed options are the best choice.

Finally, with respect to the option's sensitivity to interest rate, we know that the value of a non-indexed option is positively related to the interest rate. For options with absolute indexing, as long as t is not zero, the interest rate sensitivity is positive / negative when β is greater / less than one. For options with relative indexing, the interest rate sensitivity is positive when β is greater than one, and mostly positive when β is less than one. We omit the plots for brevity.

In summary, absolute indexing and relative indexing can create quite different incentive effects. In many ways, relative indexing leads to more desirable incentive effects. First of all, compared with absolute indexing, relative indexing is associated with a much stronger performance incentive since the option has a much bigger delta per dollar of granting cost. Second, relative indexing has a stronger risk incentive than absolute indexing due to the option's higher vega. Insofar as CEO's tend to shun risky projects due to their non-diversified personal wealth, stock options with relative indexing can be an effective tool to counter their risk-aversion and enhance the value of the firm. Third, relative indexing can better preserve the contract's value against the passage of time thanks to its smaller theta or sensitivity to time.

4. A Case Study

To illustrate how the two indexing schemes would fare in the real world, we trace the values of fictitious index options based on market data. We identify a period in which the general market experienced a cycle of upward and downward movements. Taking the Nasdaq index as the benchmark, the period from January 2, 1998 to December 31, 2001 presented such a cycle, as shown in Figure 8. The index was on the upward trend until March 2000 when it peaked at 5,049, and then

[7] We have also produced plots for other values of t including $t = 0$, and they all share the same feature.

took a downward turn. The company we select is Intel Corp. which granted 600,000 conventional stock options to its CEO, Mr. Craig R. Barrett, on January 2, 1998. These incentive options have a ten-year maturity and an exercise price equal to the stock's fair market value on the granting day. We assume away the vesting and early exercise features. In addition to the granted non-indexed options, we examine fictitious options that are indexed absolutely and relatively in the manner discussed in this paper. Using daily closes of the index and the stock for the aforementioned period, we obtain the following estimates: $\sigma_S = 0.565$, $\sigma_I = 0.379$, $\rho = 0.718$. [8] To facilitate calculations, we assume a risk-free rate of 6% and dividend yields of 0% and 2%, respectively for the stock and the index. Corresponding to the realized stock and index values, we calculate the daily theoretical option values and plot them together with the index and the stock prices. For scaling purposes, we multiply the stock price and option values by a factor of 50. Figure 8 contains the results.[9]

To begin with, we see general co-movements between the stock and index values. The relatively high correlation coefficient of 0.718 attests to this observation. This means that, most of the time, the stock's movements are attributable to the market, not the CEO's performance. In the early period after the initial grant, the option values under the two indexing schemes were indistinguishable. During the first half of 1999, Nasdaq was on an upward trend while the Intel stock headed downward. Both indexing schemes were able to reflect the relatively poor performance of the stock, as indicated by the downward trend in the option values. The same observation can be made for the later part of 1999. For the early part of 2000, both the stock and the index climbed significantly in a more or less parallel fashion. In this period, the value of the absolutely indexed option appreciated much more than its relatively indexed counterpart, reflecting its overshooting level effect. Then, when it came to the market correction in the fall of 2000, the value of the absolutely indexed option dropped much more than its relatively indexed counterpart, again indicating its overshooting level effect. Even more interesting is the period around July 2001 during which the stock hovered

[8]Daily closing prices are downloaded from http://finance.yahoo.com. The stock prices are adjusted for dividends and splits. The adjusted price for January 2, 1998 is $17.76.

[9]We also calculated the theoretical values of the non-indexed option. As expected, they are much higher than its indexed counerparts and closely mimick the trend of the stock price. Since they do not convey any interesting insights, we omit them in the plot for clarity.

around \$1450 (or \$29 per share) while the Nasdaq index was on a down turn, a case of superior performance in the relative sense. How did the two indexing schemes fare? The relatively indexed option saw a corresponding upward trend as desired, but the value of the absolutely indexed option remained more or less unchanged.

This simple case study demonstrates the drawback of absolute indexing and shows how relatively indexing can be used to correct it.

5. Conclusion

Among the many issues surrounding executive compensation, the question of whether and how to design relative performance compensation has begun to attract much attention. Johnson and Tian (2000) designed and studied an indexing scheme which enables the exercise price of the stock option to float with the market. As pointed out by Meulbroek (2001), such an indexing scheme, referred to as "absolute indexing", suffers a key drawback: it can reward CEO's for no efforts (when the stock market is on the upward trend) and penalize CEO's for potentially good efforts (when the stock market is on a downward trend). In other words, absolute indexing, by focusing only on the exercise price level, fails to address the undesirable consequence of the option's inherent property: the option's value is homogeneous of degree one with respect to the stock price and the exercise price. The option's value would increase or decrease even if the stock price has gone up or down purely because of the overall market movement, a situation that indexing is supposed to correct in the first place.

In this article, we modify the scheme by Johnson and Tian (2000) to correct the aforementioned drawback. In determining the option's payoff, rather than looking at the dollar difference between the stock price and the indexed exercise price, we examine the ratio of the stock price over the indexed exercise price, hence the term "relative indexing". We compare the two indexing schemes in terms of valuation and incentive effects.

We find that absolute indexing tends to reward too generously and penalize too harshly, especially when the firm's beta or systematic risk is high. Relative indexing can correct this deficiency and at the same time leads to many desirable incentive effects. For one thing, on a per dollar basis, it has a much higher delta, which is equivalent to a higher performance incentive. For another, relative indexing has a stronger risk incentive, which induces CEO's to undertake more risky projects which may be beneficial to the firm but otherwise avoided due to CEO's less diversified personal wealth. Finally, relatively indexed options have another advantage over non-indexed or absolutely indexed options: they can better preserve the option's value against the passage of time, i.e., they have the smallest time decay.

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[3] Meulbroek, Lisa K., 2001, “Executive Compensation Using Relative-Performance-Based Options: Evaluating the Structure and Costs of Indexes Options”, *Working Paper*, Harvard Business School.

[4] Rappaport, Alfred, 1999, “New Thinking on How to Link Executive Pay with Performance”, *Harvard Business Review,* 91-101.

Table 1. Changes in Indexed Option Values In Response to Changes in Stock Price and Index

		β =0.75			β =1.00			β =1.25		
Change in stock price (%)	Change in index level (%)	Change in benchmark price (%)	Change in option value: abs. indexing (%)	Change in option value: rel. indexing (%)	Change in benchmark price (%)	Change in option value: abs. indexing (%)	Change in option value: rel. indexing (%)	Change in benchmark price (%)	Change in option value: abs. indexing (%)	Change in option value: rel. indexing (%)
-10	-10	-7.60	-23.92	-17.66	-10.00	-10.00	0.00	-12.34	-3.89	9.63
-8	-10	-7.60	-10.47	-3.11	-10.00	-0.25	10.84	-12.34	3.57	18.15
-10	-8	-6.06	-31.83	-27.43	-8.00	-17.26	-10.06	-9.90	-10.26	-0.40
-5	-5	-3.77	-12.36	-8.93	-5.00	-5.00	0.00	-6.21	-1.88	4.61
-4	-5	-3.77	-5.39	-1.68	-5.00	-0.19	5.07	-6.21	1.75	8.49
-5	-4	-3.02	-16.68	-14.08	-4.00	-8.69	-4.89	-4.97	-5.06	-0.09
0	0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	2	1.50	5.17	3.62	2.00	2.00	0.00	2.51	0.72	-1.74
8	10	7.41	11.71	4.01	10.00	0.70	-8.45	12.65	-3.31	-14.17
10	8	5.94	37.67	29.95	8.00	17.71	8.99	10.10	9.75	-0.32
10	10	7.41	27.07	18.31	10.00	10.00	0.00	12.65	3.42	-8.19

Note: 1. This table reports changes in indexed option values in response to changes in the stock price and the index. The options were granted five years ago ($t = 5$) and have a remaining time to maturity of five years ($T - t = 5$). To avoid the complication of time-decay in option values, we assume that the stock and the index remain at their initial level ($S_0 = I_0 = \$100$) for the first five years, and then undergo a one-time change at time $t = 5$ years indicated in the first two columns. The percentage changes in the benchmark price (i.e., indexed exercise price) and the option values correspond to this one-time change.

2. We fix the correlation ρ at 0.75 and the index volatility σ_I at 0.15. For each level of β, we imply the firm's volatility σ_S.

3. Other inputs: $H_0 = \$100$, $r = 0.08$, $q_S = q_I = 0.02$, $\lambda = 1.0$.

Figure 1: Option Values vs. Stock and Index Prices (continued)

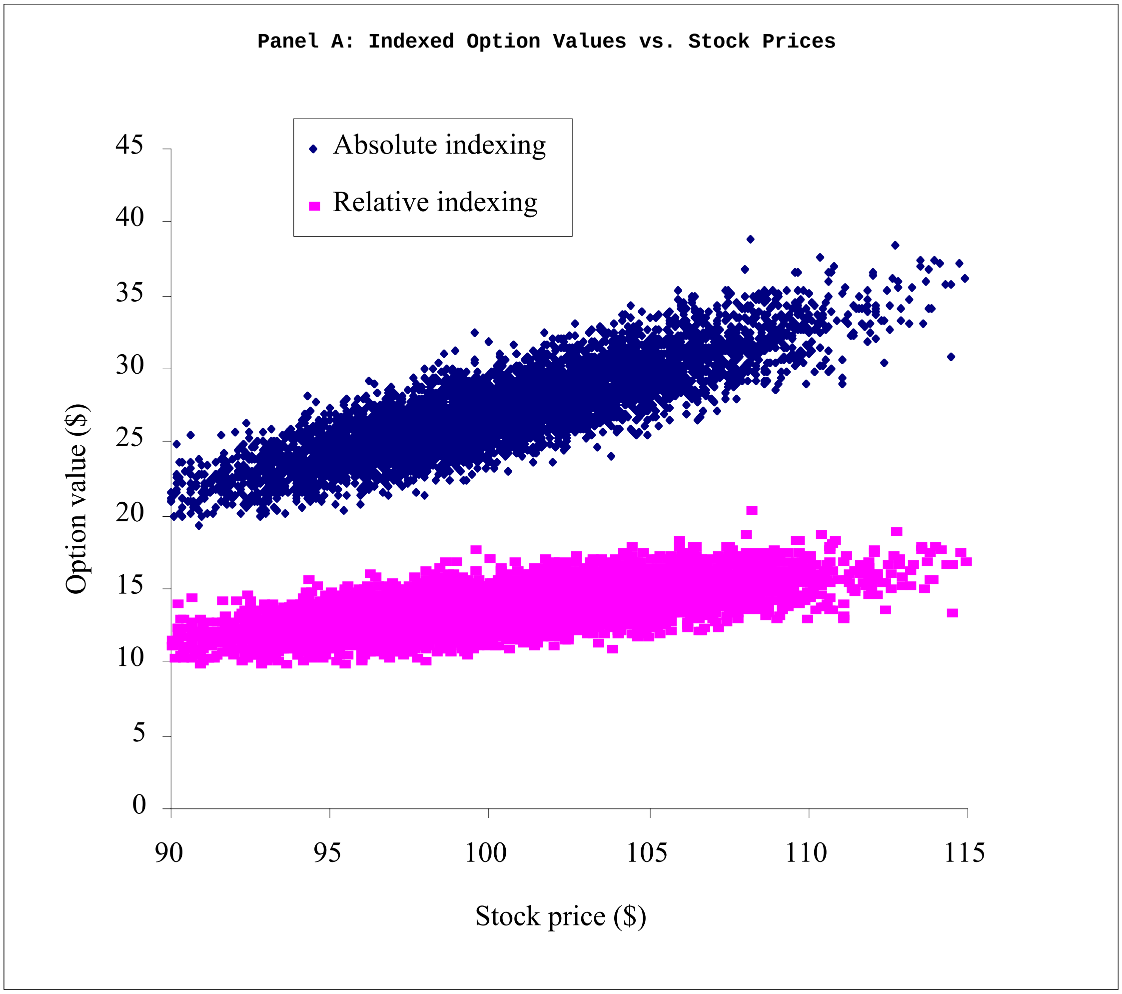
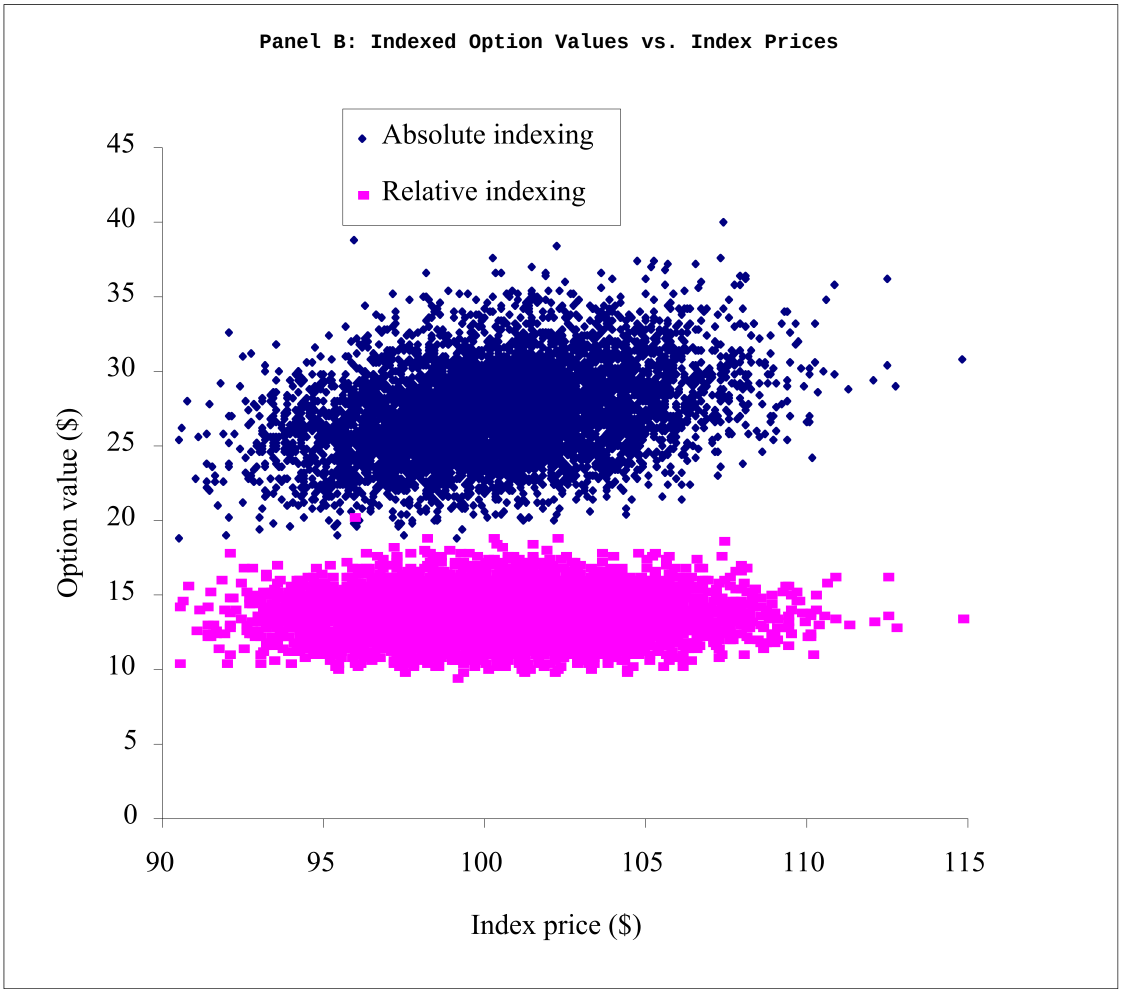


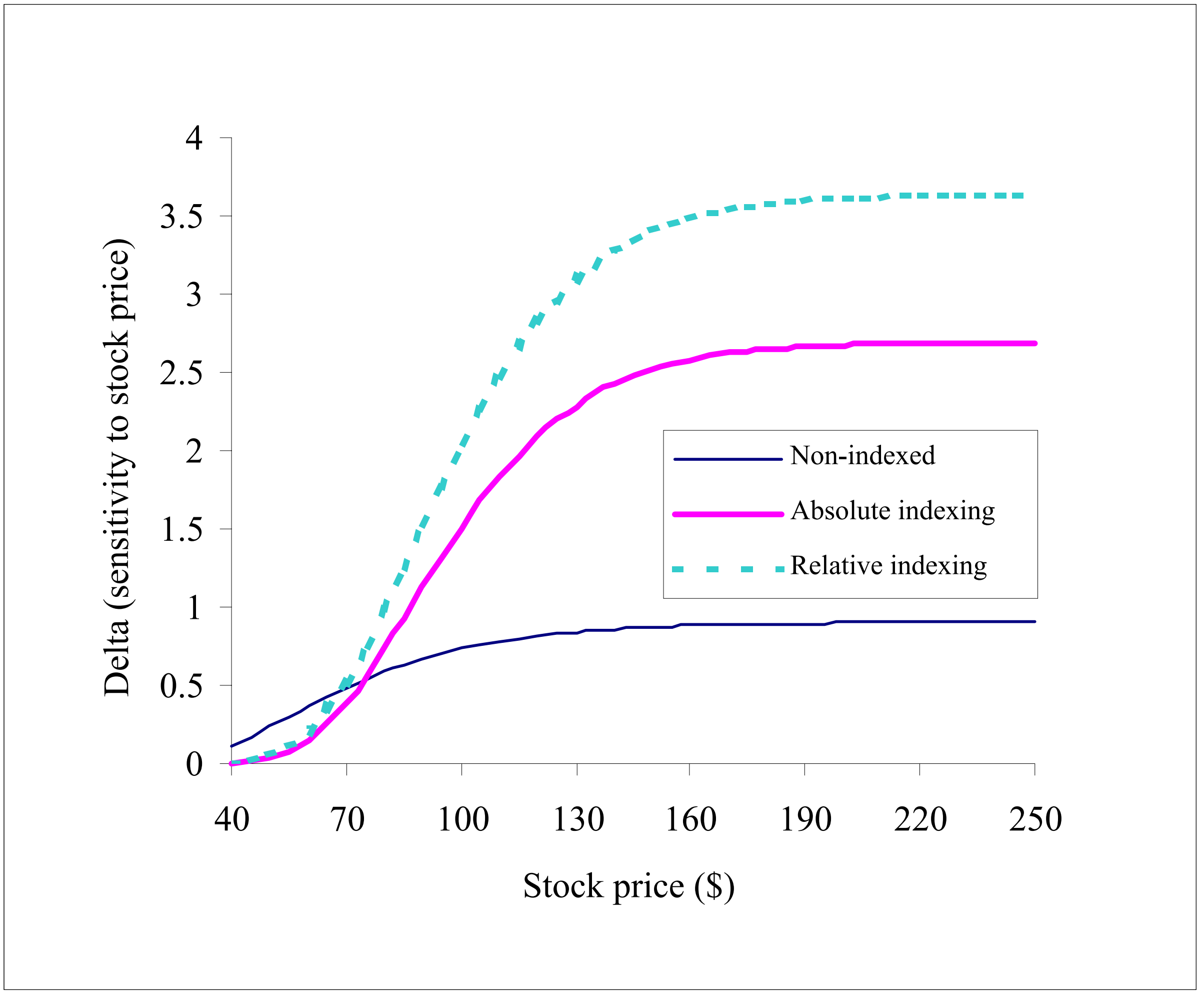
Figure 1: Option Values vs. Stock and Index Prices



Note:

1. This figure plots option values against simulated stock and index prices. The initial prices for the stock and the index are $S_0 = I_0 = \$100$. The options are 0.05 years old with a remaining time to maturity of 9.95 years (i.e., $t = 0.05$, $T - t = 9.95$). The stock price and the index price at time t are simulated by assuming the dynamics in (2.1) and (2.2), while assuming $\mu_S = r + \beta(\mu_I - r)$. The option values are calculated based on the realized S_t and I_t. Each panel plots 5,000 realizations.
2. For better visual effects, we multiply the value of the absolutely indexed option by 1.5.
3. Other inputs for calculations: $H_0 = \$100$, $\sigma_S = 0.20$, $\sigma_I = 0.15$, $\rho = 0.75$, $r = 0.08$, $q_S = q_I = 0.02$, $\mu_I = 0.12$, $\lambda = 1.0$.

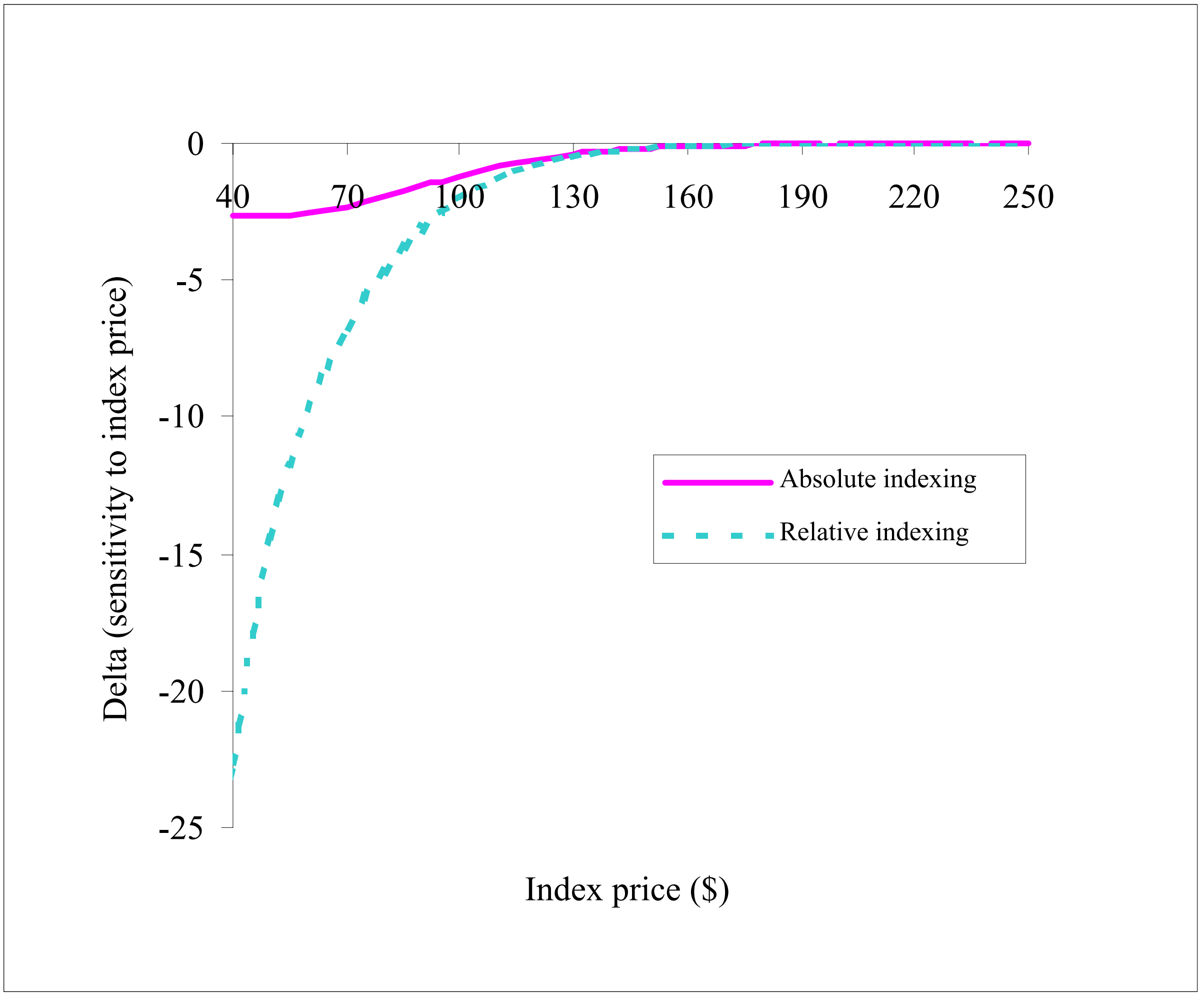
Figure 2: Option's Stock Price Delta vs. Stock Price



Note:

1. This figure plots option's sensitivity to the stock price against the stock price itself. The index level is fixed at $100. The options are five years old with a remaining time to maturity of five years (i.e., $t = 5$, $T - t = 5$). To ensure comparability, we adjust the number of indexed options so that all options have the same value initially. These adjustment factors are then applied to the delta's of the indexed options.
2. Other inputs for calculations: $S_0 = I_t = I_0 = H_0 = \$100$, $\sigma_S = 0.20$, $\sigma_I = 0.15$, $\rho = 0.75$, $r = 0.08$, $q_S = q_I = 0.02$, $\lambda = 1.0$.

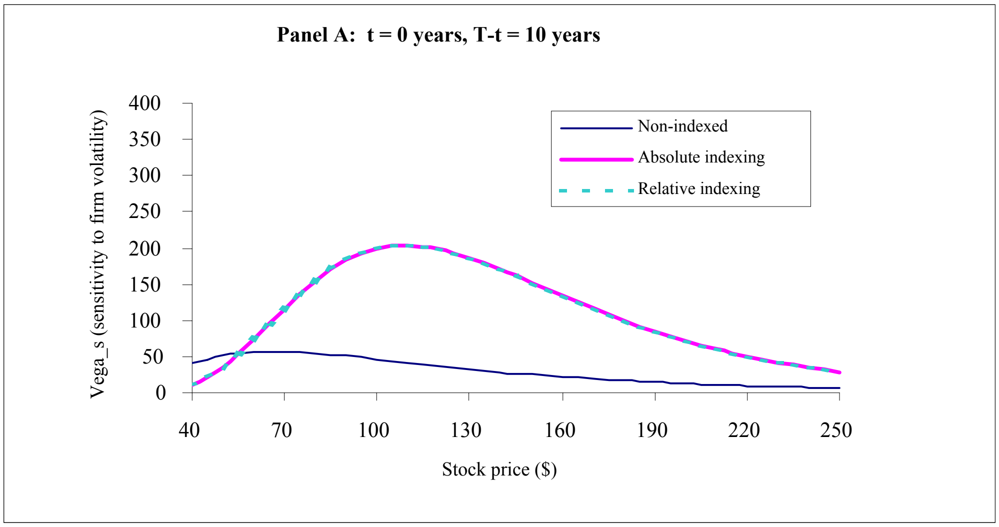
Figure 3: Option's Index Price Delta vs. Index Price

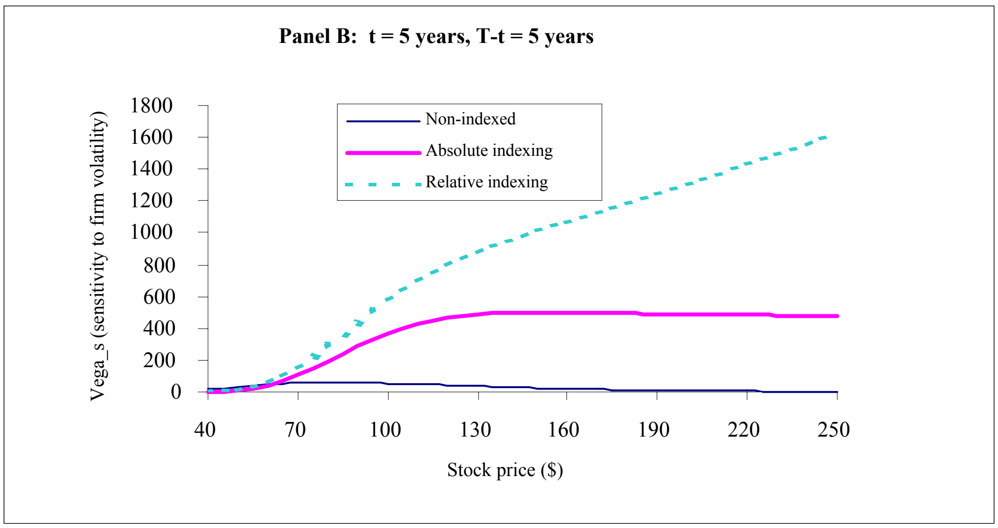


Note:

1. This figure plots option's sensitivity to the index price against the index price itself. The stock price is fixed at \$100. The options are five years old with a remaining time to maturity of five years (i.e., $t = 5$, $T - t = 5$). To ensure comparability, we adjust the number of indexed options so that all options have the same value initially. These adjustment factors are then applied to the delta's of the indexed options.
2. Other inputs for calculations: $S_0 = S_t = I_0 = H_0 = \$100$, $\sigma_S = 0.20$, $\sigma_I = 0.15$, $\rho = 0.75$, $r = 0.08$, $q_S = q_I = 0.02$, $\lambda = 1.0$.

Figure 4: Option's Firm Volatility Vega vs. Stock Prices

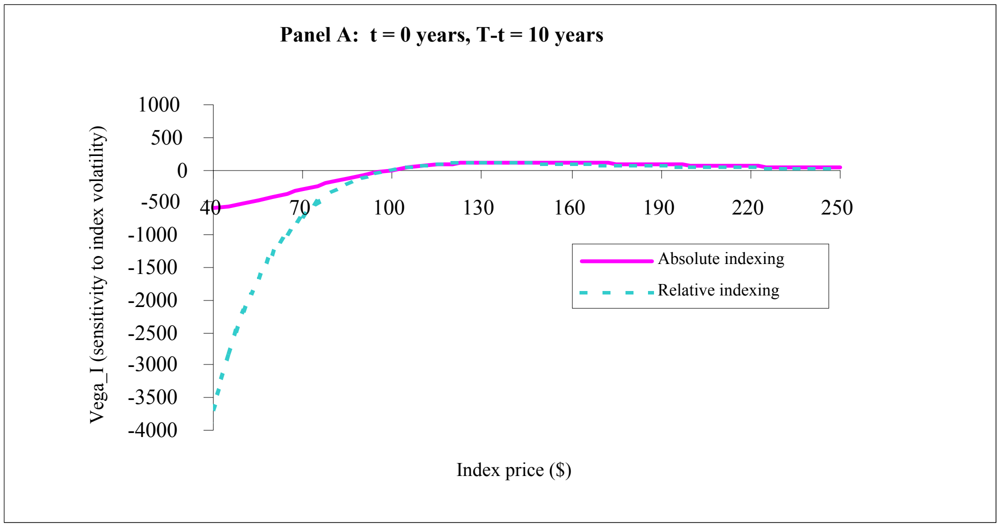


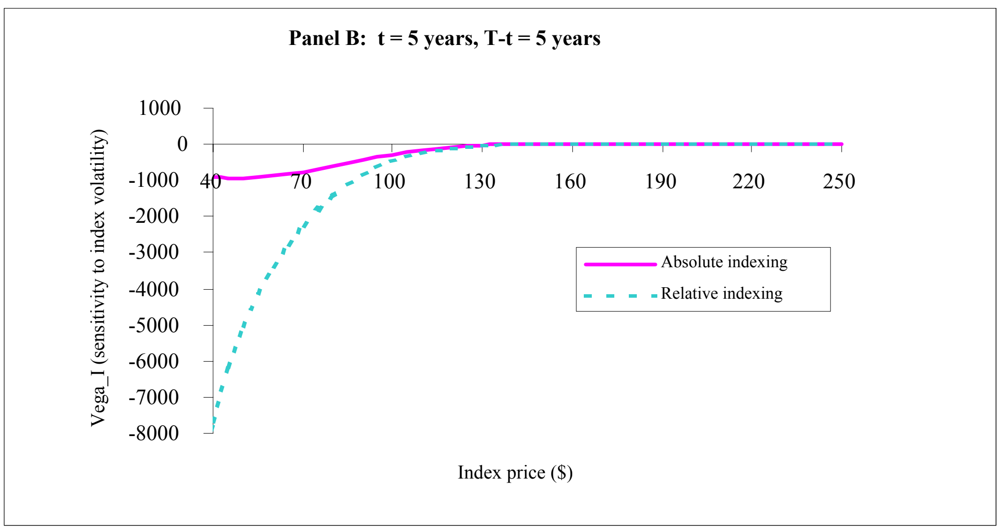


Note:

1. This figure plots option's firm volatility vega against the stock price. The index level is fixed at \$100. Options in Panel A are newly granted (i.e., $t = 0$, $T - t = 10$); options in Panel B are five years old with a remaining time to maturity of five years (i.e., $t = 5$, $T - t = 5$). To ensure comparability, we adjust the number of indexed options at each level of firm volatility σ_S so that all options have the same value initially. These adjustment factors are then applied to the vega's of the indexed options.
2. Other inputs for calculations: $S_0 = I_t = I_0 = H_0 = \$100$, $\sigma_S = 0.25$, $\sigma_I = 0.15$, $\rho = 0.75$, $r = 0.08$, $q_S = q_I = 0.02$, $\lambda = 1.0$.

Figure 5: Option's Index Volatility Vega vs. Index Prices

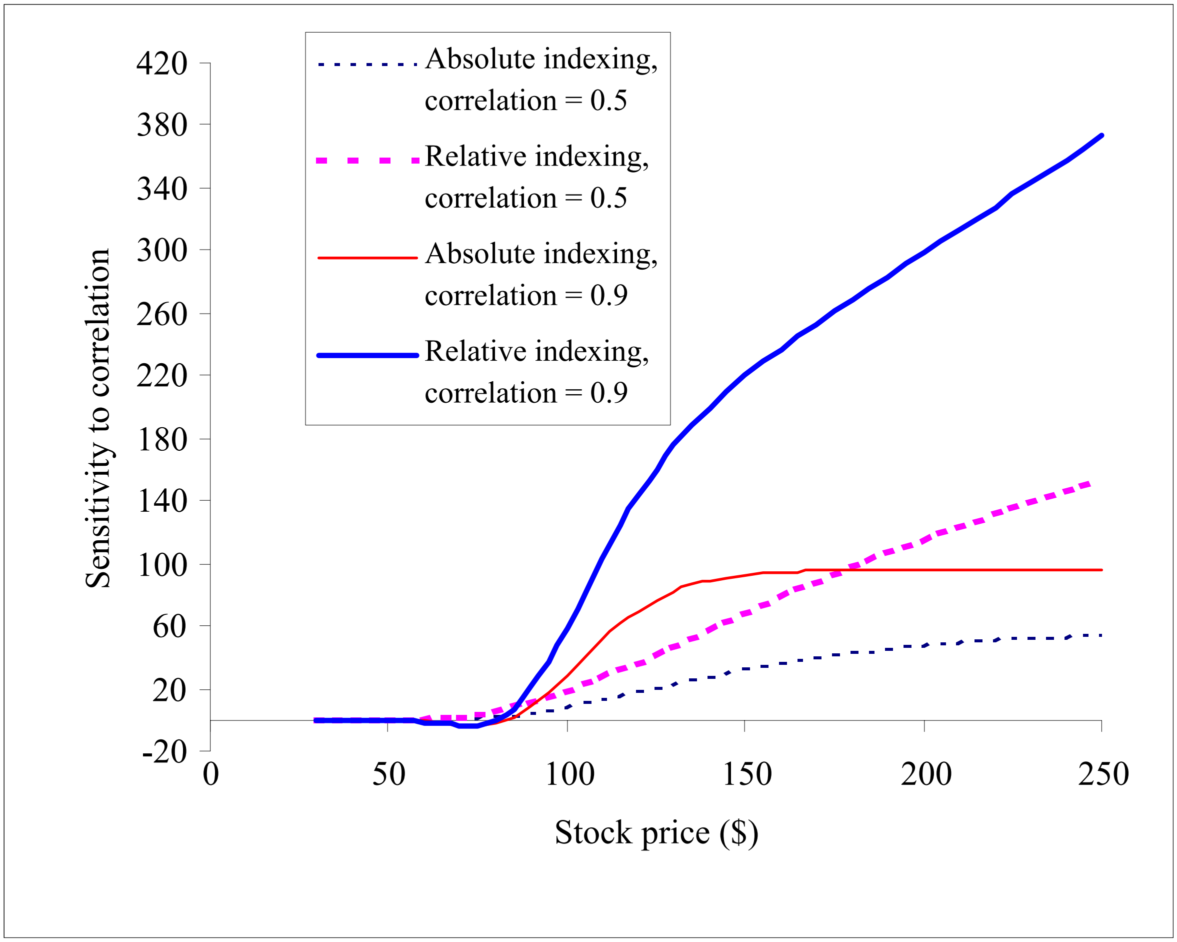




Note:

1. This figure plots option's index volatility vega against the index price. The stock price is fixed at \$100. Options in Panel A are newly granted (i.e., $t = 0$, $T - t = 10$); options in Panel B are five years old with a remaining time to maturity of five years (i.e., $t = 5$, $T - t = 5$). To ensure comparability, we adjust the number of indexed options at each level of index volatility σ_I so that all options have the same value initially. These adjustment factors are then applied to the vega's of the indexed options.
2. Other inputs for calculations: $S_0 = S_t = I_0 = H_0 = \$100$, $\sigma_S = 0.25$, $\sigma_I = 0.15$, $\rho = 0.75$, $r = 0.08$, $q_S = q_I = 0.02$, $\lambda = 1.0$.

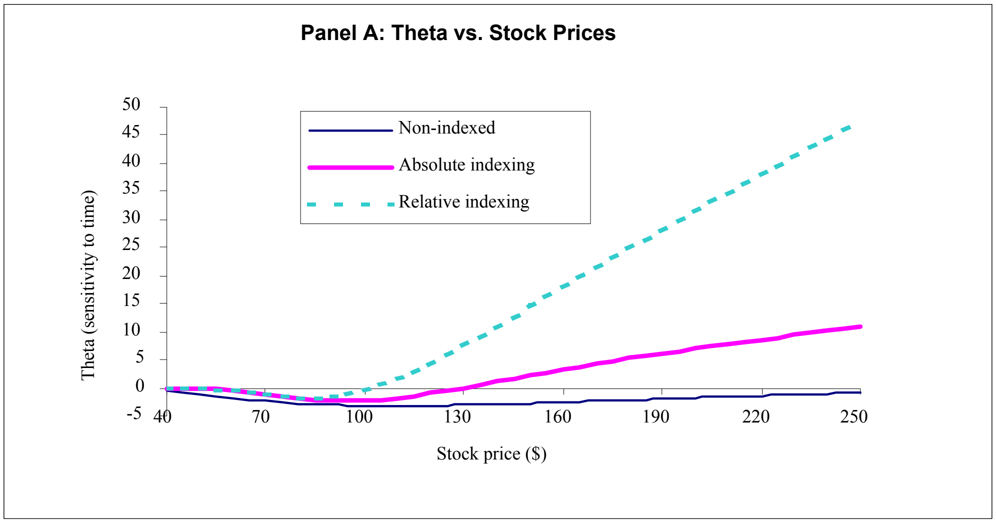
Figure 6: Indexed Option's Sensitivity to Correlation vs. Stock Prices

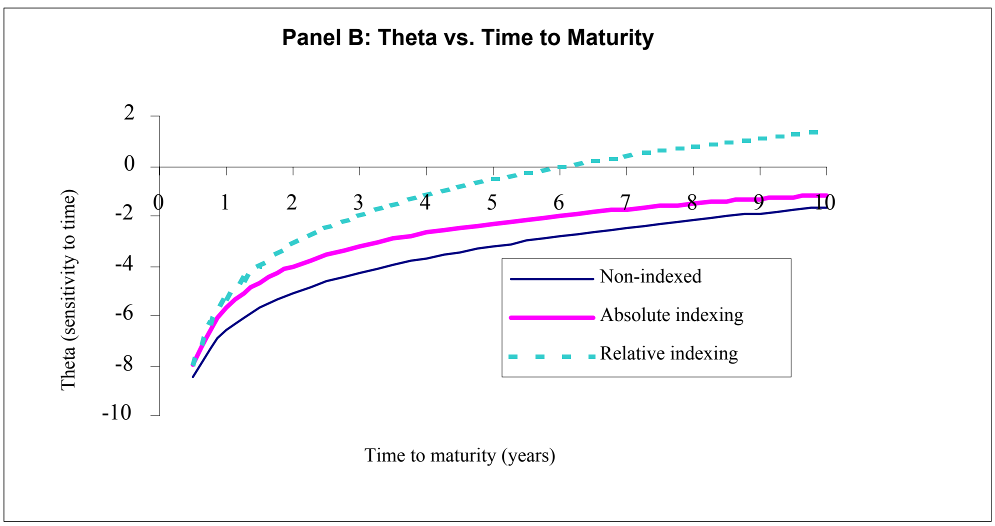


Note:

1. This figure plots indexed option's sensitivity to correlation against the stock price All options are five years old with a remaining maturity of five years (i.e., $t = 5$, $T - t = 5$).
2. Other inputs for calculations: $S_0 = I_t = I_0 = H_0 = \$100$, $\sigma_S = 0.20$, $\sigma_I = 0.20$, $\rho = 0.75$, $r = 0.08$, $q_S = q_I = 0.02$, $\lambda = 1.0$.

Figure 7: Option's Theta (Sensitivity to Time) vs. Stock Prices and Maturities

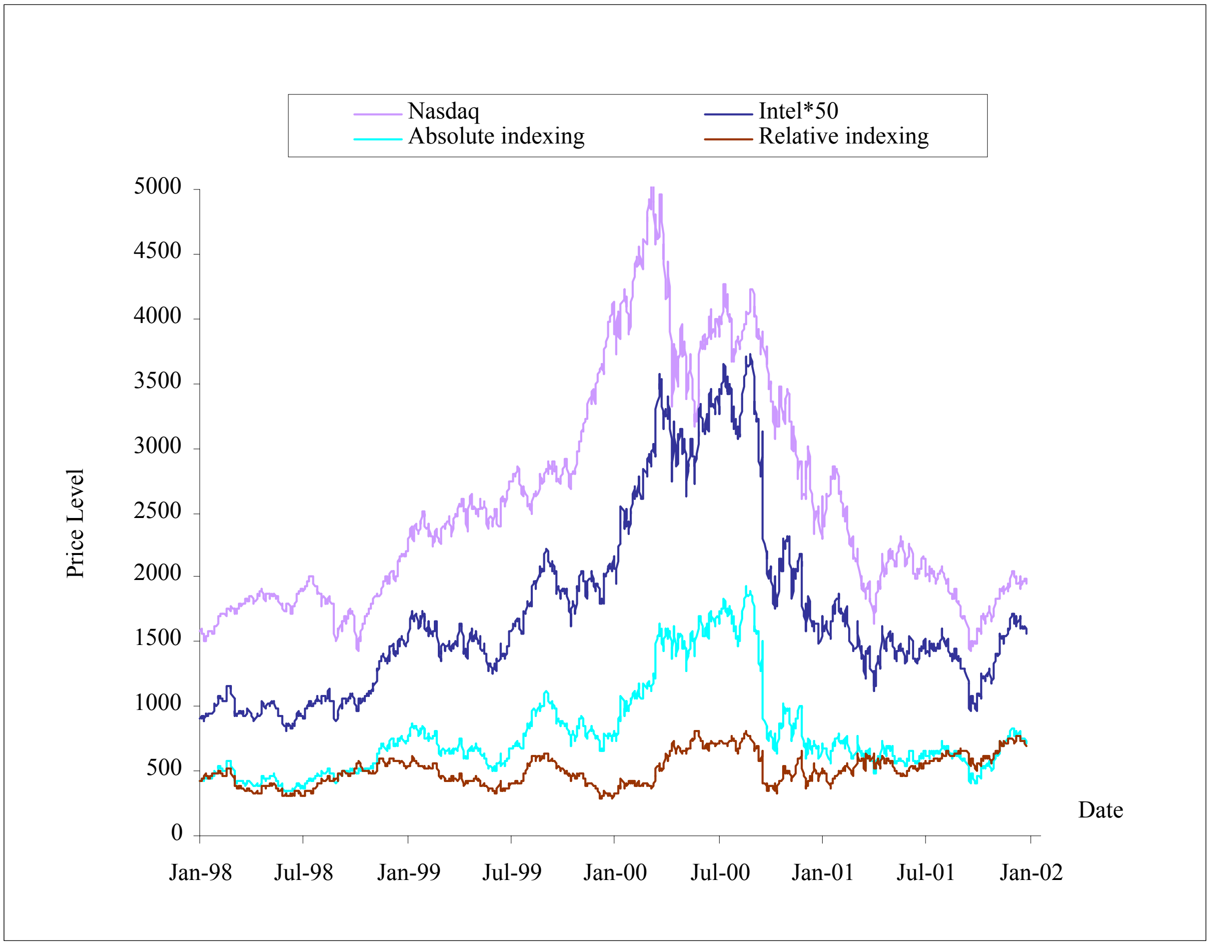




Note:

1. This figure plots option's theta (sensitivity to time) against stock prices and maturity. In Panel A, we assume that all options are five years old with a remaining time to maturity of five years (i.e., $t = 5$, $T - t = 5$). To ensure comparability, we adjust the number of indexed options so that all options have the same value initially at $t = 0$. In Panel B, all options are one year old, with a remaining to maturity being plotted (i.e., $t = 1$, $T - t = 1, 2, 3...$). Again, we adjust the number of indexed options so that all options have the same value initially at $t = 0$ with a time to maturity $T - t = 2, 3, 4...$ .These adjustment factors are then applied to the indexed options' theta for each S_t or $T - t$.
2. Other inputs for calculations: $S_0 = I_t = I_0 = A = \$100$, $\sigma_S = 0.20$, $\sigma_I = 0.20$, $\rho = 0.75$, $r = 0.08$, $q_S = q_I = 0.02$, $\lambda = 1.0$.

Figure 8: Tracing the Option Values for Intel Corp's CEO



Note:

1. This figure plots daily closes of the Nasdaq index and the Intel stock price multiplied by 50 (the upper two graphs). It also plots the daily values of indexed options (absolute and relative indexing) from Jan. 2, 1998 to December 31, 2001 (the lower two graphs). Again, for scaling purposes, the option values are multiplied by 50. We are tracing two contracts granted on Jan. 2, 1998. The stock price on the granting date was \$17.76 (aftetr adjusting for dividends and splits according to http://finance.yahoo.com). All options were granted at the money. The CEO, Mr. Craig R. Barrett, was granted 600,000 units of stock options with a time to maturity of 10 years. We assume away vesting and early exercise features. Indexing based on the Nasdaq index is fictitious.
2. The following parameter estimates are obtained using daily closes of the index and the stock: $\sigma_S = 0.565$, $\sigma_I = 0.379$, $\rho = 0.718$. We assume a constant interest rate of 6%, a zero dividend yield for the stock, and a 2% dividend yield for the index.