Stress Testing with a Bottom-Up Corporate Default Prediction Model[*]

Jin-Chuan Duan[†], Weimin Miao[‡] and Tao Wang[§]

(First Draft: November 4, 2012; This Draft: August 5, 2014)

Abstract

We propose a credit stress testing method that puts together: (1) a bottom-up corporate default prediction model capable of translating shocks to its input variables into the default likelihood of a target portfolio, and (2) a set of stress-testing regressions which relate the presumed adverse macroeconomic scenarios to shocks to the inputs of the default prediction model. This method can produce distributions of corporate defaults for any target portfolio (economy or sector) over various horizons of interest. A set of seven macroeconomic variables are used to define stress scenarios, and these seven variables are found to explain a substantial fraction of variations in the input variables of the corporate default prediction model used in our analysis. Our back-testing empirical study focuses on (1) the ASEAN 5 countries because the experience of the 1997 Asian financial crisis makes this group ideal for an out-of-sample performance study over the 2008-09 financial crisis period, and (2) the US and Eurozone for its wider applicability. We also consider the IMF-style V-shaped and protracted stress recovery scenarios to demonstrate our method's practicality.

Keywords: risk management, stress testing, credit risk, default prediction, forward intensity.

1 Introduction

Risks are ever present and can appear in an idiosyncratic fashion destroying some businesses or individuals with negligible impact on the whole economy. Sometimes, risks get manifested in the form of financial crisis with large-scale and wide-ranging disruptions to an economy, a region or the whole world. Risk management has long been understood to be critical to the long-run survival of a business and/or the smooth functioning of an economy. The 2008-09 financial crisis has, however, proven that risk management at either the individual or system level has not been adequate. That crisis, the subsequent Eurozone sovereign debt turmoil and the ensuing policy responses in the form of Quantitative Easing continue to reverberate to this date. Naturally, they have seriously challenged the conventional wisdom and prompted all sorts of opinions/recommendations/solutions for going forward. This paper adds to the literature by proposing a credit stress testing method that can help identify vulnerability in the financial system and risk inherent to a credit portfolio.

Risk management is a multi-facet undertaking with stress testing being one increasingly important component. As opposed to analysing and managing day-to-day normal variations caused by many external and internal factors, stress testing addresses uncommon risk events and zeros in on the extreme circumstances that are expected to happen only rarely. The banking sectors in many jurisdictions are already required to conduct stress testing to determine whether their financial systems are sound. However, stress testing can be useful only if the contemplated stress scenarios are properly conceived and the system/model that is used to translate stress scenarios into outcomes is adequate. The solution proposed in this paper is predicated on coupling two models. One of which focuses on translating, via a set of stress-testing regressions, the prescribed macroeconomic stress scenarios into shocks to the input variables of a corporate default predication model. The other is a corporate default prediction model that turns the shocks to its inputs into a distribution of corporate defaults for an economy and/or some target portfolio of interest over an intended horizon. We then employ this method to study stress outcomes in a backtesting fashion by using a reasonable set of stress variables and treating realized outcomes on these variables as stress scenarios beforehand. In addition, we apply the method while adopting the IMF-style V-shaped and protracted stress recovery scenarios to show its applicability for practical regulatory usage.

Stress testing has received considerable attention in the literature during the past few years. Central banks and international organizations have taken the leading role in this area of research due to their specific concerns for financial stability. Several survey studies are available which include Blaschke, *et al* (2001), Burrows, *et al* (2012), Jones, *et al* (2004), Sorge (2004), Moretti, *et al* (2008), and Foglia (2009). IMF (2012) and Jobst, *et al* (2013) discussed the principles and practices of the IMF's Financial Sector Assessment Program (FSAP). Hoggarth, *et al* (2005) described a number of approaches and the results of stress testing carried out in the UK as part of the FSAP. Worrell (2004) discussed an integrated approach which incorporates stress testing, early warning systems and financial soundness indicators. Allen and Saunders (2004) provided surveys on the pro-cyclicality of credit and market risk measures.

Most of the studies attempt to develop credit stress testing methods by linking macroeconomic conditions to different measures of credit risk. The choice of credit risk measures usually depends

on data availability. The credit risk models of Lehmann and Manz (2006) and van den End, *et al* (2006) used the loan loss provisions (LLP) ratio to measure credit quality at the individual bank level. However, the use of LLP ratio restricts the credit stress testing to the banking sector and may not even adequately reflect credit risk of that sector. Several papers including Virolainen (2004), van den End, *et al* (2006) and Coletti, *et al* (2007) employed historical default rates as the credit risk measure to build models in the framework of Wilson (1997a&b). Similar to the LLP ratio, realized default rates are a retrospective indicator of credit risk and can also substantially differ from the true default rates that they are meant to reflect.[1] Naturally, such measure is unlikely to serve as a good proxy for credit risk under stress because of its noisy and non-forward-looking nature. Asberg and Shahnazarian (2008), Castren, *et al* (2009), Chan-Lau (2013) and Ferry, *et al* (2012a&b) adopted Moody's KMV expected default frequencies (EDFs) to model the average credit quality of listed companies. The EDF is a forward-looking, market-based measure of credit risk that gauges a firm's probability of default. In the spirit of using a forward-looking measure, our approach is in line with those of using EDFs.

In this paper, we employ probabilities of default (PDs) generated from a bottom-up, reduced-form credit risk model. For credit stress testing, the preferable model is the bottom-up type, meaning that the model can be used to predict individual firm defaults and to aggregate individual PDs to the default profile of a portfolio.[2] Modelling directly at the portfolio level (i.e., a top-down approach) will often run into many practical difficulties; for example, a portfolio of investment-grade firms is unlikely to experience any meaningful number of default cases, making it difficult to establish a reliable statistical link between the portfolio's default profile and the variables that are used to define stress scenarios. We opt for the forward-intensity corporate default prediction model of Duan, *et al* (2012) because it can conveniently and consistently generate term structures of default probabilities for individual obligors and for easy aggregation. As explained in Duan, *et al* (2012), their forward-intensity approach allow users to bypass the challenging task of specifying and estimating a very high-dimensional auxiliary system needed for generating future values of obligors' attributes critical to getting PDs of future periods. The forward-intensity approach thus makes modelling a large sample of obligors and predicting defaults over longer horizons operationally feasible. It should be noted that because PDs are forward-looking, our stress testing output is the future default prospect rather than the actual default trajectory.

Our model choice also reflects the availability of the credit research infrastructure created by the Credit Research Initiative (CRI) of the Risk Management Institute (RMI) at the National University of Singapore. The current RMI-CRI default prediction model is based on the forward-intensity default prediction approach of Duan, *et al* (2012). The RMI-CRI model employed in this

---

[1]Assume that the true default rate for a company over one year is 80%. It may turn out that the company has actually survived the period, reflecting an event with only a 20% of chance. In short, realized default rates can be quite a noisy measure.

[2]It should be noted that our bottom-up approach, i.e., aggregating individual firms up to a portfolio, will be considered a top-down approach according to the terminology of the IMF bank solvency stress testing (IMF (2012) and Jobst, *et al* (2013)), which classifies stress testing conducted by individual banks using their internal models as bottom-up whereas those conducted by the IMF treating individual banks as elements of a portfolio as top-down.

paper is Version 2013, Update 2b with Addenda 1-4. As of December 2013, the RMI-CRI generates daily updated PDs for around 34,000 currently active, exchange-listed firms in 106 economies in Asia-Pacific, Europe, Americas, Middle East and Africa. In addition, historical time series of PDs for the delisted firms (bankruptcies, M&As, etc) in these economies are also available, making the total number of exchange-listed firms in excess of 60,000. For the three regions (ASEAN 5, the US and Eurozone 12) on which we perform stress testing, their average 1-month (12-month) accuracy ratios, typically used in measuring prediction performance, are 77.5% (68.0%) for ASEAN 5, 93.8% (81.5%) for the US, and 85.4% (71.8%) for Eurozone 12, respectively.[3] The RMI-CRI system adopts an open platform, making public all technical details, and the access to the results is free. Therefore, our proposed credit stress testing method can easily be operationalized to generate frequently updated stress testing results by simply leveraging the RMI-CRI infrastructure.

The forward-intensity model of Duan, *et al* (2012) as implemented by the RMI-CRI generates term structures of PDs for individual firms and by implication portfolios using two macroeconomic variables and six firm-specific attributes. One quick way to conduct stress testing is to directly link the PDs generated from the default prediction model to the stress variables. A more fundamental approach is to relate the input variables of the default prediction model to the macroeconomic variables that define the stress scenarios.[4] One can first stress the macroeconomic conditions to predict the corresponding changes to the model's inputs, and then compute the resulting default profile of the target portfolio. The change in the default profile will then reflect how vulnerable the target portfolio is under stress. In this paper, we adopt the latter approach.

We relate each of the input variables of the default prediction model (i.e., two macroeconomic variables and the six firm-specific attributes) to a set of stress variables via a stress-testing regression. The stress variables are all treated as contemporaneous regressors, but other control variables, if any, will enter on the lagged basis. Such regressions are not meant for forecasting; rather they are used to determine the impact on the default prediction model's input variables by assuming that the stress variables have changed by a certain magnitude. Specifically, we run the stress-testing regression for each of the six firm-specific variables on sector-by-sector basis; for example, we calculate the sector averages for, say, distance-to-default, and treat the average value as the dependent variable in the stress-testing regression for the distance-to-default. We choose not to run the stress-testing regressions on the individual-firm basis for two reasons. First, many firms lack a long time series of data to allow for a stable stress-testing regression result. Second, it would introduce a large number of stress-testing regression parameters, and the sampling errors of the stress-testing regression could then be propagated to the stressed PDs. In order to translate a sector stress-testing regression prediction to the prediction on a firm in that sector, we simply maintain the firm's position relative to the average under stress for each of its six firm-specific attributes

---

[3]The accuracy ratios are taken from Table B.1 of the RMI-CRI Technical Report (2013). This table only reports accuracy ratios for economies with more than 20 defaults in the testing set. Therefore, we calculate the average accuracy ratio for each region using only economies included in this table.

[4]It should be noted that the macroeconomic variables used to define the stress scenarios may differ from the two macroeconomic variables used by the forward intensity model. To avoid confusion, we refer to the macroeconomic variables defining stress scenarios as stress variables.

to be identical to its current relative position. However, this assumption could be relaxed in real applications if firm-specific knowledge is available, because it could be factored into the generation of more sensible stress testing output.

Our empirical analysis focuses on three regions: ASEAN 5, the US and Eurozone 12. ASEAN 5 includes Indonesia, Malaysia, the Philippines, Singapore and Thailand whereas Eurozone 12 includes Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain. Our sample period is January 1990 to December 2013. Our choice of these three samples reflects the fact that ASEAN 5 countries went through two large-scale credit crises during the sample period of January 1990 to December 2013, i.e., the 1997 Asian financial crisis and the 2008-09 global financial crisis, whereas the US and Eurozone only experienced one credit crisis that started in 2008. The two experiences contrast nicely in terms of how well the stress testing method performs, and allow us to draw some reasonable conjecture.

To represent different hypothetical adverse economic scenarios, we include as stress variables a set of macroeconomic variables such as real GDP growth rate, change in unemployment rate, inflation rate, exchange rate, etc., which are commonly used in the literature of stress testing. In addition, we also consider commodity price index and the US equity market volatility index because different economies are highly integrated and dependent on the global financial conditions. The global financial conditions are expected to exert direct and significant impact on individual economies and eventually the overall creditworthiness of their corporate sectors.

An important aspect in assessing a stress testing model's specification, as pointed out in Foglia (2009), is the overall in-sample and out-of-sample performances. "A common feature of macro-econometric models of credit risk is that macroeconomic variables alone tend to explain a fairly small part of the variation of the dependent variable, especially when only one or two macroeconomic variables are considered." said Foglia (2009). By including seven stress variables in our analysis, we show that our stress testing method performs reasonably well both in-sample and out-of-sample. The R^2 for the stress-testing regressions over a one-year horizon are usually high (around 40%), suggesting that a substantial fraction of variation in the input variables of the default prediction model can be explained by the set of stress variables employed.

We compare both the predicted PDs (peeking into the macroeconomic environment one year ahead) with the actual PDs a year later generated by the RMI-CRI system using the realized input values. We perform this analysis for the three regions over the sample period. For each region, we consider three groups: all firms, financial firms only, and non-financial firms. An economy may or may not have previously experienced a credit crisis to reasonably estimate the stress-testing regressions; for example, the ASEAN countries went through the 1997 Asian financial crisis which presumably allows for a more reliable prediction on the credit risk response during the 2008-09 global financial crisis. Our credit stress testing results in the case of ASEAN 5 show good performance in the sense that the method is able to pick up the heightened credit risk profile under severe economic stress of the 2008-09 global financial crisis. In the case of the US and Eurozone 12 during our sample period and prior to the 2008-09 global financial crisis, there was no credit

crisis to train the stress-testing regression equations. Thus, the equations trained to the data prior to the global financial crisis is unlikely to capture the magnitude of shocks to the input variables of the default prediction model if one contemplates a stress scenario like the global financial crisis before it actually occurred. Not surprising, the stress testing performance for the US and Eurozone are not as good as that for ASEAN 5. However, if we train the stress-testing regressions to the whole sample including the global financial crisis and then use them for stress testing, performance becomes much better. Our results suggest that the credit stress testing method proposed in this paper is likely to work for the US and Eurozone for future credit crises, because the 2008-09 global financial crisis has provided valuable data that help train the stress-testing regressions.

In applications, policy makers devise various stress scenarios to test resilience of the financial system. We show that the IMF-style V-shaped and protracted stress recovery scenarios can be easily implemented with our stress testing method. Stress is defined only in terms of real GDP growth rates in some driving economies, and is characterized by a fall of one or two standard deviations below the baseline projections for the first year. These economies are then tracked for next five years in their prescribed return toward the baseline, faster for the V-shaped recovery and slower for the protracted recovery. All other stress variables need to move in a consistent manner, which is accomplished in our analysis by running a structural vector autoregression on the real GDP growth rates of the driving economies along with other stress variables, which contemporaneously depend on the real GDP growth rates but not the other way around. Both the V-shaped and the protracted stress recovery analyses are conducted on the financial sector of ASEAN 5, the US and Eurozone 12 at two time points – March 2009 and December 2013. The results show that the US financial sector would have kicked up substantially its already very high level of credit risk, had the economy followed the V-shaped or protracted stress recovery scenario from March 2009 onwards. For ASEAN 5 in March 2009, the V-shaped or protracted stress recovery scenario would have pushed up another notch their relatively high credit risk level, although not as pronounced as those for the US. In the case of Eurozone 12, the impact would be rather minor except for the effect caused by a more pessimistic baseline projection. For the endpoint of our sample period (December 2013), however, the V-shaped or protracted stress recovery scenario would only have marginal adverse effect for all three groups, indicating that their financial sectors were much sounder after emerging out of the 2008-09 financial crisis.

2 Stress Testing Methodology

2.1 Default Prediction Model

The bottom-up corporate default prediction model used in this study is the forward-intensity model of Duan, *et al* (2012) being implemented by the Risk Management Institute of National University of Singapore on 106 economies around the world as part of its public-good Credit Research Initiative. Duan, *et al* (2012) modelled the occurrence of defaults/bankruptcies and other types of firm exits such as mergers and acquisitions using two Poisson processes with stochastic intensities. The two Poisson processes are independent conditional on stochastic intensities. The forward intensities for default/bankruptcy are functions of some state variables, also known as covariates, with each

Table 1: Covariates employed in the default prediction model

Covariate	Description
Stock index (EQTY)	Trailing 1-year return on the stock market
Short-term risk-free rate (TBILL)	Yield on 3-month government bills
Distance-to-default (DTD)	Each firms distance-to-default, which is a volatility adjusted leverage measure based on Merton (1974)
Cash/TA (LIQ)	Ratio of each firms sum of cash and short-term investments to total assets
NI/TA (PROF)	Ratio of each firms net income to total assets
Relative size (RSIZE)	Logarithm of the ratio of each firms market capitalization to the economys median market capitalization
Market-to-book value (MB)	Ratio of each firms market value (market capitalization plus total book value of liabilities) to its book value (total book value of assets)
Idiosyncratic volatility (SIGMA)	1-year idiosyncratic volatility of each firm, computed as the standard deviation of its residuals using the market model

This table lists the two macroeconomic risk factors and the six firm-specific attributes used in the default prediction model. The data are monthly sampled up-to-date observations over the period of January 1990 to December 2013. The specific model implemented is the RMI-CRI system (Version 2013, Update 2b with Addenda 1-4). The model uses "level" for all six variables, but only uses "trend" for DTD, LIQ, PROF and RSIZE. "Level" is calculated as the twelve-month moving average and "trend" is the deviation of the current value from "level". For further details, readers are referred to the RMI-CRI Technical Report (2013).

function corresponding to a forward starting time. Similarly, there is a set of forward intensity functions for other exits. State variables influence default and other exits, but there is no feedback loop. The forward intensities can then be used to compute PD for any future period of interest.

The PD of firm i at time t for the prediction horizon of $(t, t + \tau]$ can be generically expressed as

$$p_{i,t}(\tau) = P_\tau(\mathbf{X}_t, \mathbf{Y}_{i,t}) \tag{1}$$

where $P_\tau(\cdot)$ denotes the PD function for horizon τ, $\mathbf{X}_t$ denotes the common risk factors at time t collectively, and $\mathbf{Y}_{i,t}$ denotes the firm-specific attributes of firm i at time t collectively. The model is implemented with one month as a basic period. For further technical details, readers are referred to the RMI-CRI Technical Report (2013).

Table 1 lists the set of two macroeconomic and six firm-specific variables that characterise the forward intensity functions, four of which are deployed in terms of both level and trend. In total, there are twelve state variables for all forward intensity functions. This implies that the credit risk of a corporate borrower can arise from three sources: its fundamental risk as reflected in its financial statements; market perception of its fundamental risk as reflected in its stock price movements; and

common economic risk factors which define the market condition and in turn affect the well-beings of individual firms.

2.2 Stress-Testing Regressions

To evaluate the impact of a given stress scenario on corporate default probabilities, one needs to rely on the historical relationship between the input variables of the default prediction model and the macroeconomic stress variables describing the adverse economic scenarios. Such relationships can for instance be established using a linear regression model by also incorporating its own lagged values. Adverse macroeconomic scenarios are basically some presumed future trajectory of the macroeconomic stress variables. The stress-testing regression in essence provides a sensible dynamic evolution around the presumed trajectory for the input variables of the default prediction model. For the common risk factors of the default prediction model, we run the stress-testing regression in an obvious way. For the firm-specific attributes, however, we perform the stress-testing regression with the dependance variable being the country-specific industry averages of each of these attributes instead of running on individual firms. This is a reasonable way to dampen sampling errors. We will describe later how a predicted industry average under stress is turned into the predicted individual attributes for different firms.

Define $\bar{Y}_{i,j,t}$ as the i-th country-industry average of the j-th firm-specific variable, $X_{m,t}$ be the m-th element of $\mathbf{X}_t$, and $Z_{k,t}$ as the k-th macroeconomic stress variable at month t. Let $\Delta X_{m,t}$ and $\Delta \bar{Y}_{i,j,t}$ be the one-period difference of $X_{m,t}$ and $\bar{Y}_{i,j,t}$, respectively. Note that the set of stress variables may or may not contain the common risk factors of the default prediction model. In general, our stress-testing regression model is defined in the following equations:

$$\Delta X_{m,t} = \beta_{m,0}^X + \sum_{k=1}^{n} \beta_{m,k}^X Z_{k,t} + \gamma_{m,1}^X X_{m,t-1} + \gamma_{m,2}^X X_{m,t-2} + \varepsilon_{m,t}^X, \tag{2}$$

$$\Delta \bar{Y}_{i,j,t} = \beta_{i,j,0}^Y + \sum_{k=1}^{n} \beta_{i,j,k}^Y Z_{k,t} + \gamma_{i,j,1}^Y \bar{Y}_{i,j,t-1} + \gamma_{i,j,2}^Y \bar{Y}_{i,j,t-2} + \varepsilon_{i,j,t}^Y, \tag{3}$$

where $m = 1, 2$ and $j = 1, 2, \cdots, 6$. Note that we have used two lagged values in the above regressions because our empirical analysis shows importance of using two lags.

The stress-testing regressions can be better understood with an example. Suppose that one wants to predict the financial sector's distance-to-default under the presumed scenario defined by the GDP and unemployment rate monthly over next year. We regress the past observations of monthly difference of the industry average distance-to-default on their corresponding observations of the GDP monthly growth rate and the monthly change in unemployment rates. In addition, we take the distance-to-default lagged by one and two months as additional regressors to account for autocorrelation. These monthly stress-testing regression functions can then be used to create 12 future predicted values of $X_{m,t}$ and $\bar{Y}_{i,j,t}$ consistent with the presumed trajectories of GDP and unemployment rate over next year. In our later implementation, the stress-testing regressions are conducted on a monthly basis using monthly data series. The error terms $\varepsilon_{m,t}^X$ and $\varepsilon_{i,j,t}^Y$ are

assumed to be independent and identically distributed over time, respectively. We observe that correlations among the residuals of the country-specific industry averages are rather weak across different attributes for the same industry, but strong for the same attribute across different industries. Therefore for our implementation, we only account for the correlations across industries.

Later in the empirical analysis, we will conduct stress testing monthly which calls for running the stress-testing regressions monthly and on monthly data. Although the input variables of the default prediction model are available monthly, macroeconomic series such as GDP are only available quarterly. So, we need to devise a sensible way to deal with mixed-frequency data in estimating equations (2)-(3). Note that our mixed-frequency regression problem is one with the explanatory variables being less frequently available, which fundamentally differs from, say, what the mixed-data sampling (MIDAS) regression in Ghysels, *et al* (2010) sets out to address. Our solution presented in Appendix A is to deduce from equation (2) or (3) an time-aggregated form so that missing values for the explanatory variables in a higher frequency no longer materially influence the estimation results.

2.3 Stress Variables and Scenarios

Drawing from the recent experience in industry-wide stress testing of banks in different jurisdictions, we settle on seven stress variables that are commonly used and could affect operating revenue, operating cost, or funding cost of firms. A stress scenario can be defined by the whole set of stress variables or just a subset of one or a few variables. When more than one stress variable is used to define a scenario, it is critical to ensure that the scenario is plausibly defined by several variables, which can, for example, rely on a macroeconometric model already employed a policy maker or a structural vector autoregression to ensure internal consistency.

The seven stress variables employed in this paper are provided in Table 2. Real GDP reflects the state of an economy with its growth rate serving as a proxy for the growth in incomes and earnings of firms. A higher growth would generally lead to higher corporate earnings and lower default risk. Unemployment rate affects the consumption and spending of households. An increase in the unemployment rate would generally result in lower revenues for firms, particularly those that are consumer-oriented (e.g., restaurants, retailers, etc.), and increase their default likelihoods. Consumer price index provides a measure of inflation and controlling inflation rates is one of the primary objectives of monetary policy. High inflation is usually considered a signal of macroeconomic mismanagement and a source of uncertainty. Higher inflation leads to increased costs and tends to impair credit quality. However, higher inflation may also reduces debt burden in real term, thereby improves creditworthiness. Exchange rate affects the bottom-line of a wide range of firms directly or indirectly through trade and investment flows; for example, a stronger domestic currency will typically benefit importers and likely hurt exporters. Firms in a small open economy will be particularly sensitive to exchange rate movements. We use the BIS nominal effective exchange rate indices, which are calculated as geometric weighted averages of bilateral exchange rates.[5] Short-term floating rate is a common benchmark that banks use in determining lending

---

[5]For more information, please see http://www.bis.org/statistics/eer/index.htm.

Table 2: Macroeconomic variables used in defining stress scenarios

Macro variables	Description	Period used
Real GDP growth rate (GDP)	Percentage change of seasonally adjusted real gross domestic product	Quarterly (1990Q1–2013Q4)
Unemployment rate (UNEMP)	Difference of seasonally adjusted unemployment rate	Quarterly (1990Q1–2013Q4)
Consumer price index (INFL)	Percentage change of consumer price index	Monthly (1990M1–2013M12)
Exchange rate (NEER)	Percentage change of the BIS nominal effective exchange rate	Monthly (1990M1–2013M12)
Short-term floating rate (IBOR)	Difference of 3-month interbank offer rate	Monthly (1990M1–2013M12)
Commodity price index (GSCI)	Percentage change of the S&P GSCI commodity index	Monthly (1990M1–2013M12)
Market volatility index (VIX)	Percentage change of the CBOE VIX	Monthly (1990M1–2013M12)

This table lists the seven macroeconomic stress variables employed in the empirical analysis. All data are retrieved from Datastream, Bank for International Settlements (BIS) and Chicago Board Options Exchange (CBOE) on July 20, 2014. The unemployment rates in Indonesia, Malaysia, Philippines, Thailand are only available in a non-seasonally adjusted form. The consumer price index is only available in a non-seasonally adjusted form expect for Singapore, France, Germany and the US.

rates for variable-rate loans. It is also the rate that is applicable to short-term corporate funding via debt/commercial paper markets. A higher borrowing rate/cost of funding is expected to increase default risk of firms owing to higher interest expense.[6] The S&P GSCI commodity index captures an important production factor cost. A higher commodity price typically benefits commodity producers but causes deteriorated creditworthiness for other companies. The VIX index offered by the Chicago Board Options Exchange reflects the volatility in the S&P 500 index portfolio and is a commonly used proxy variable for gauging the risk level in US stock markets, but can also be indicative of how volatile stock markets in other economies, given the global linkage of financial markets and dominance of the US economy.

Each stress scenario contemplated at time t is represented by a set of presumed trajectories of the macroeconomic stress variables over a forward period $(t, t + l]$, denoted by $\{\tilde{Z}_{k,t+j} : k = 1, \cdots, n; j = 1, \cdots, l\}$. In our backtesting study later, l is set to 12, i.e., one year, whereas for the IMF-style V-shaped and protracted stress recovery analyses, l is set to 72, i.e., six years. Under a specific stress scenario, the corresponding changes to the common risk factors and the industry average of firm-specific variables can be simulated and advanced forward monthly using equations (2) and (3). Simulation will be repeated many times under the same presumed scenario to obtain

---

[6]The long-term interest rate would also be useful, particularly for corporate debt, but was excluded because the availability of long-term benchmark yields is limited for some economies.

statistics of interest.

The predicted industry average under stress for each firm-specific variable in a country/economy needs to be converted to individual firm values before inserting them into the default prediction model. For the j-th attribute in the i-th industry, we compute the current difference between a firm's value for the j-th attribute and the i-th industry average of the j-th attribute, and then add the difference to the predicted i-th industry average for the j-th attribute. In essence, we assume that the value of a firm's attribute relative to its industry average in that country/economy remains unchanged under the stress scenario. One could argue that such an assumption is too restrictive, and running the stress-testing regression on the individual firm basis may be a better alternative. However, running such regressions on the individual firm basis will create too many parameters and introduce large sampling errors into the stress testing system. In fact, our experience shows that running stress-testing regressions on individual firms would have a rather poor performance, consistent with the intuition. Of course, if one has a strong prior on how a particular firm would perform under the economy-wide stress vis-à-vis the industry average, such a prior could be easily incorporated.

3   Back Testing the Stress Testing Method

3.1   Data

In our backtesting study, ASEAN 5 (Indonesia, Malaysia, Philippines, Singapore and Thailand), the US and Eurozone 12 (Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain) are used to illustrate the stress testing method. Our PD related data are retrieved from the Credit Research Initiative of Risk Management Institute, National University of Singapore, and are based on the RMI-CRI January 2014 calibration in which the data period is up to the end of 2013. Our sample data set contains all public firms in these three regions over the period from January 1990 to December 2013. Our choice of these three regions reflects the fact that ASEAN countries have gone through two large-scale credit crises during this sample period – the 1997 Asian financial crisis and the 2008-09 global financial crisis – whereas the US and the Eurozone countries have only experienced one major credit crisis that started in 2008. The two types of experience contrast nicely in terms of how well the stress testing method performs, which in turn allow us to draw some reasonable conjecture. The seven macroeconomic stress variables defining stress scenarios along with their sources are described in Table 2.

Since the default prediction model implemented under the RMI-CRI uses winsorized input values, we adopt the same winsorization criterion for the stress-testing regressions where each of the six firm-specific attributes is winsorized at the 0.5 and 99.5 percentiles. Companies are classified into ten industries according to Bloomberg's industry classification. These are basic material, communications, consumer (cyclical), consumer (noncyclical), diversified, energy, financial, industrial, technology and utilities. In our ASEAN 5 sample, there are in total 6,276 firms, among which 1,410 companies belong to the financial industry and the other 4,866 firms are non-financial. In the case of the US, our sample comprises 15,408 firms, of which 3,023 are financial, 12,263 are non-financial

Table 3: Average in-sample 12-month R^2 for stress-testing regressions: ASEAN 5

Sectors	Input Variables of the Default prediction model					
	DTD	LIQ	PROF	RSIZE	MB	SIGMA
Basic material	35.61%	37.41%	36.60%	25.42%	40.82%	36.80%
Communications	39.83%	44.21%	39.44%	26.84%	47.92%	34.22%
Consumer (cyclical)	41.07%	27.60%	37.87%	26.27%	40.28%	37.25%
Consumer (noncyclical)	37.46%	31.83%	42.64%	34.30%	39.57%	40.47%
Diversified	31.07%	37.99%	35.67%	40.79%	45.05%	41.25%
Energy	41.71%	30.27%	43.82%	33.78%	47.14%	38.35%
Financial	37.00%	14.56%	39.30%	32.36%	48.20%	42.45%
Industrial	38.20%	28.44%	36.07%	25.99%	45.13%	38.96%
Technology	35.55%	28.65%	39.75%	24.22%	39.54%	35.90%
Utilities	36.98%	39.60%	40.49%	47.08%	52.11%	39.53%

This table reports the average (over 5 countries) in-sample 12-month R^2 for stress-testing regressions as in equation (3). Firms are classified into 10 industries based on Bloomberg's industry classification. DTD is a firm's distance to default, LIQ is the ratio of cash and short-term investments to total assets, PROF is the ratio of net income to total assets, RSIZE is the logarithm of the ratio of each firm's market capitalization to the economy's median market capitalization. MB is the market to book ratio and SIGMA is the idiosyncratic volatility. The in-sample 12-month R^2 for each country is first calculated and then averaged across five countries.

and 122 are not classified. For the Eurozone 12 sample, there are altogether 11,810 firms, among which 2,938 are financial, 8,090 are non-financial and 782 are not classified. The companies that are not classified by Bloomberg will be assigned industry averages that are the means of the corresponding ten industry averages of a given country/economy. Industry averages are calculated as the 20% (both directions) trimmed mean for robustness.

3.2 Back Testing by Treating the Actuals as the Scenarios

3.2.1 Back testing analysis: ASEAN 5

In this section, we report the performance of our stress testing method on the ASEAN 5 countries. The in-sample 12-month R^2 for our stress-testing regressions are reported in Table 3.[7] For most of the industries considered, the reasonably good R^2 suggest that sector averages of the firm-specific attributes move in sync with business cycle and a significant fraction of the variation in these variables are explained by the seven stress variables. The stress-testing regression of the macro risk factors, i.e., equation (2), yields an average 12-month R^2 of 63.56% for the trailing 1-year return of stock market and 55.84% for the 3-month government bill rate.

For stress testing at the end of the sample period, the above in-sample stress-testing regressions

---

[7]The method for calculating the 12-month R^2 while using monthly data is explained in Appendix A.

suffice because all data were actually available then. To conduct a back testing analysis of credit stress testing using the realized economic environments, however, one can only use the data sample available at the time of testing. We opt for an expanding window approach that uses all data accumulated up to the time of stress testing to perform the stress-testing regressions (2) and (3). The estimated regression equations are then used to generate future values for the input variables of the default prediction model, and follow by deducing the credit risk profile under the stress testing assumption that presumes the subsequent realized economic environment were anticipated one year earlier. Doing so, the stress testing study becomes a legitimate out-of-sample analysis.

We focus our analysis on three target groups/portfolios because of their general interest. The first portfolio consists of all firms in our ASEAN 5 data set, whereas the second and the third are portfolios for all financial firms and non-financial firms, respectively. The RMI-CRI default prediction model used in our analysis can generate term structures of credit risk, i.e., PDs for different horizons. Here, we only report the results based on the 1-month and 12-month PDs with the first reflecting the immediate credit outlook under the stress scenario, whereas the second reflecting the intermediate vulnerability for the firms in the portfolio.

Starting from January 2002 and every month afterwards until December 2012, we first estimate regressions (2) and (3) using the data in the expanding window. We then simulate the input variables of the default prediction model 12 months later using the estimated equations (2) and (3). The presumed stress scenario used in the simulations are the actual realizations of these variables over next 12 months. Because GDP and unemployment rate are available on a quarterly basis, the monthly values are linearly interpolated within each quarter. For four out of the six firm-specific attributes, both their level and trend are needed by the default prediction model. We therefore use the simulated paths of these four variables to compute their level and trend components. For each simulation, we calculate the median PDs for different target groups and use them as our measure of a portfolio's creditworthiness. We repeat the simulation 1,000 times to obtain the mean value of the portfolio's median PDs and use it as our stress testing output. The resulting average medians at different stress testing time points are then compared with the portfolio's actual median PDs 12 months later that are generated by the default prediction model using the subsequently realized input values.

Figures 1 and 2 report the 1-month and 12-month PDs, respectively. For each figure, we plot the stress testing outputs (average median of the simulated 1-month or 12-month PDs in the group) and compare them with the actual median PDs generated by the default prediction model one year later. For a meaningful comparison, we remove at each testing time the firms that were delisted within next 12 months in the calculation of the median of the stress testing outputs. We delay the stress testing PDs for one year to align them with the actual PDs that only become available one year later. Since our data sample ends in December 2013, the stress testing exercise has to end in December 2012 so as to align with the actual PD in December 2013. For both figures, the stress testing outputs (dashed curve) move in tandem with the actual PDs (solid curve), suggesting that our stress testing method, as far as picking up heightened credit risk is concerned, works well for the ASEAN 5 countries.

13

Figure 1: Back testing analysis of ASEAN 5: 1-month PDs using the expanding window stress-testing regressions

The median values of the stressed 1-month PDs for three groups/portfolios over the sample period are reported (dashed curve). At the end of each month starting from January 2002, we re-estimate regressions (2) and (3) using an expanding window with data up to that time. The input values for the default prediction model are simulated (1,000 runs) for individual firms based on equations (2) and (3) by assuming a stress economic scenario that was realized 12 months later. For each simulation, we compute the portfolio-median PD for a target group. We then use the mean of the 1,000 simulation runs as the final median value for the group. Also plotted for comparison is the actual median PDs (solid curve) generated by the default prediction model using the realized input values 12 months later.
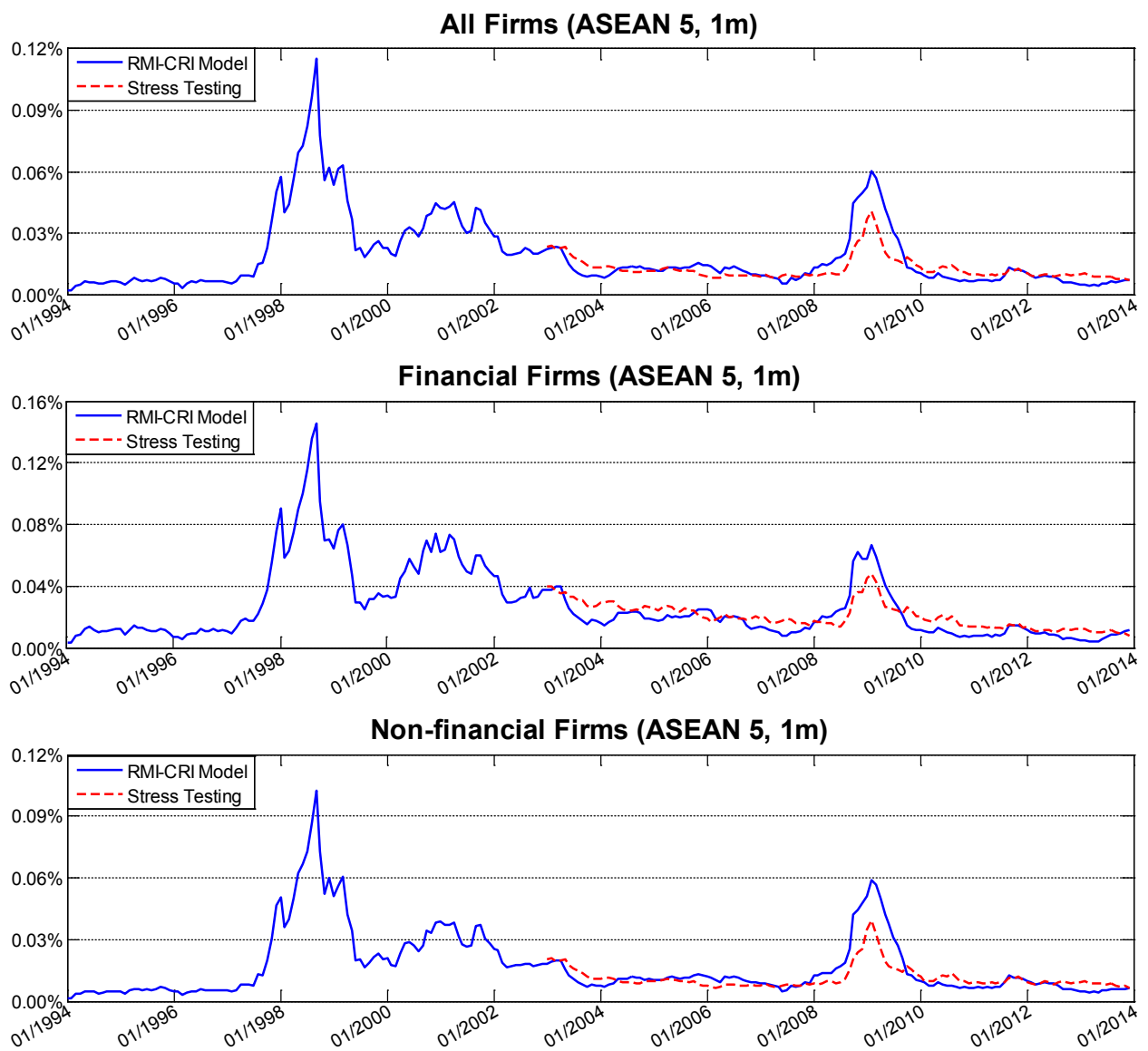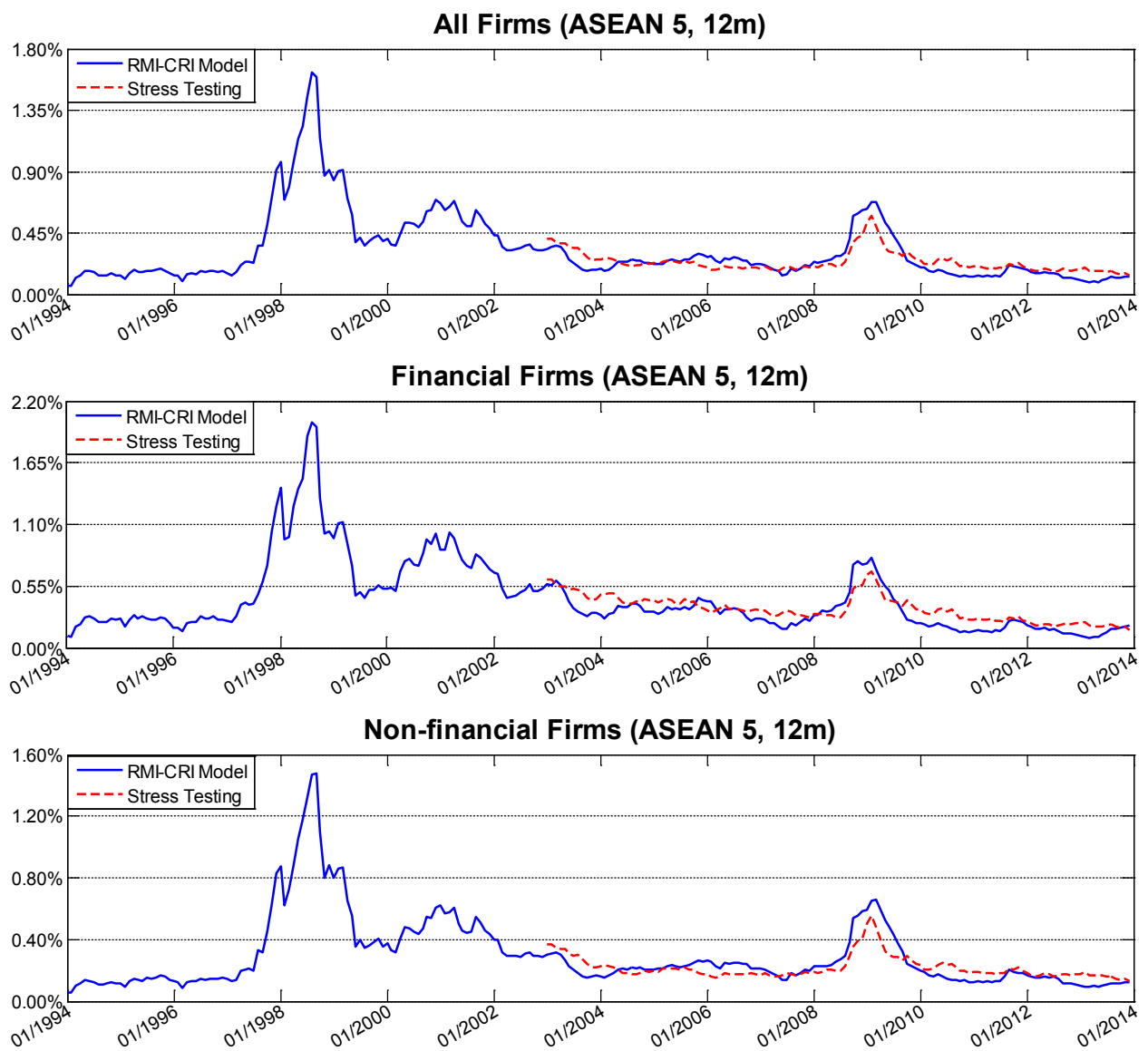
Figure 2: Back testing analysis of ASEAN 5: 12-month PDs using the expanding window stress-testing regressions

The median values of the stressed 12-month PDs for three groups/portfolios over the sample period are reported (dashed curve). At the end of each month starting from January 2002, we re-estimate regressions (2) and (3) using an expanding window with data up to that time. The input values for the default prediction model are simulated (1,000 runs) for individual firms based on equations (2) and (3) by assuming a stress economic scenario that was realized 12 months later. For each simulation, we compute the portfolio-median PD for a target group. We then use the mean of the 1,000 simulation runs as the final median value for the group. Also plotted for comparison is the actual median PDs (solid curve) generated by the default prediction model using the realized input values 12 months later.

15

3.2.2 Back testing analysis: US and Eurozone 12

In this section, we report the performance of our stress testing model on the US and Eurozone 12. The in-sample 12-month R^2 for stress-testing regressions are reported in Tables 4-5. The R^2 performance measures for individual attributes in either the US or Eurozone 12 are comparable to that of ASEAN 5. The stress-testing regression of the macro risk factors, i.e., equation (2), yields in the case of the US an average 12-month R^2 of 62.01% for the trailing 1-year return of stock market and 38.87% for the 3-month government bill rate. For Eurozone 12, the R^2 for the two macro risk factors are 58.34% and 39.92%, respectively.

We conduct a similar expanding-window analysis on the US and European firms and find less than satisfactory performance because the surge in PDs over the 2008-09 financial crisis cannot be picked up. This finding is not at all surprising, however, knowing that there was no credit crisis over our sample period prior to the 2008-09 financial crisis. The earlier internet bubble in the US, for example, has significant impact on the high-tech sector but quite limited influence on other industries of the economy. Thus, we switch to using the whole sample for the stress-testing regressions to ascertain this conjecture, knowing fully well that this approach could not have been implemented prior to the 2008-09 financial crisis. The rest of the stress testing analysis follows exactly as in the expanding-window approach.

The results reported in Figures 3 and 4 show that the stress testing method indeed works as long as the stress-testing regressions are trained to the data with a credit crisis. Note that we have skipped the 1-month PDs plots to conserve space but the conclusion is similar to that of the 12-month PDs. We thus conjecture that our stress testing method is likely to work for the US and Eurozone countries going forward, because these economies had already experienced a significant credit crisis (the global financial crisis of 2008-09) for training the stress-testing regressions.

4   Stress Testing under the IMF-style Stress Recovery Scenario

We now show how our stress testing method can be applied under the IMF-style V-shaped and protracted stress recovery scenarios. For the illustrative purpose, stress testing is conducted for the financial sector of ASEAN 5, the US and Eurozone 12, respectively. We only employ the three IMF-selected stress variables – real GDP growth rate, change in unemployment rate and inflation rate – to run the stress-testing regressions (2) and (3). Performance is also good with the average in-sample R^2 ranging from 20.60% to 59.80% for ASEAN 5, from 13.78% to 59.37% for the US, and from 29.02% to 55.33% for Eurozone 12.

In our analysis, both the IMF-style V-shaped and protracted stress recovery scenarios are assumed to be driven by the real GDP growth rate of one or several driving economies.[8] The driving economies for ASEAN 5 are assumed to be the real GDP growth rates of all the economies

---

[8]Note that the driving economies need not be in the set of the stress testing economies; for example, one can study stress in the ASEAN 5 countries with a stressed US real GDP growth.

Table 4: In-sample 12-month R^2 for stress-testing regressions: the US

Sectors	Input Variables of the Default prediction model					
	DTD	LIQ	PROF	RSIZE	MB	SIGMA
Basic material	50.18%	2.04%	14.02%	20.53%	54.89%	42.69%
Communications	46.33%	15.26%	17.32%	35.74%	47.46%	20.62%
Consumer (cyclical)	43.38%	2.50%	43.65%	11.02%	52.07%	39.62%
Consumer (noncyclical)	38.11%	19.24%	18.29%	16.37%	53.18%	24.51%
Diversified	24.22%	25.32%	53.28%	35.73%	47.38%	15.34%
Energy	53.35%	26.08%	39.11%	36.94%	59.26%	40.13%
Financial	33.44%	46.27%	38.99%	15.23%	42.65%	50.18%
Industrial	52.67%	3.87%	13.32%	15.86%	57.34%	32.71%
Technology	44.31%	12.46%	21.65%	23.76%	52.72%	15.66%
Utilities	37.00%	1.92%	10.11%	18.66%	38.74%	41.15%

This table reports the average in-sample 12-month R^2 for stress-testing regressions as in equation (3). Firms are classified into 10 industries based on Bloomberg's industry classification. DTD is a firm's distance to default, LIQ is the ratio of cash and short-term investments to total assets, PROF is the ratio of net income to total assets, RSIZE is the logarithm of the ratio of each firm's market capitalization to the economy's median market capitalization. MB is the market to book ratio and SIGMA is the idiosyncratic volatility.

Table 5: Average in-sample 12-month R^2 for stress-testing regressions: Eurozone 12

Sectors	Input Variables of the Default prediction model					
	DTD	LIQ	PROF	RSIZE	MB	SIGMA
Basic material	38.66%	36.68%	34.32%	30.57%	40.00%	41.00%
Communications	36.12%	34.31%	26.77%	28.64%	33.65%	28.45%
Consumer (cyclical)	33.03%	30.65%	27.07%	34.95%	36.81%	34.98%
Consumer (noncyclical)	36.85%	28.35%	25.85%	31.06%	34.77%	34.57%
Diversified	39.05%	37.82%	34.85%	43.57%	39.61%	42.21%
Energy	43.00%	36.40%	36.40%	24.49%	43.08%	46.04%
Financial	31.28%	38.43%	33.15%	35.02%	35.71%	38.80%
Industrial	31.33%	32.59%	34.22%	28.78%	38.97%	39.09%
Technology	41.46%	36.39%	35.19%	26.05%	35.28%	37.61%
Utilities	40.08%	27.95%	25.59%	27.68%	33.41%	43.29%

This table reports the average (over 12 countries) in-sample 12-month R^2 for stress-testing regressions as in equation (3). Firms are classified into 10 industries based on Bloomberg's industry classification. DTD is a firm's distance to default, LIQ is the ratio of cash and short-term investments to total assets, PROF is the ratio of net income to total assets, RSIZE is the logarithm of the ratio of each firm's market capitalization to the economy's median market capitalization. MB is the market to book ratio and SIGMA is the idiosyncratic volatility. The in-sample 12-month R^2 for each country is first calculated and then averaged across twelve countries.

Figure 3: Back testing analysis of the US: 12-month PDs using the whole-sample stress-testing regressions

The median values of the stressed 12-month PDs for three groups/portfolios over the sample period are reported (dashed curve). At the end of each month starting from January 2002, we run stress testing by always using regressions (2) and (3) that are estimated using the whole sample. The input values for the default prediction model are simulated (1,000 runs) for individual firms based on equations (2) and (3) by assuming a stress economic scenario that was realized 12 months later. For each simulation, we compute the portfolio-median PD for a target group. We then use the mean of the 1,000 simulation runs as the final median value for the group. Also plotted for comparison is the actual median PDs (solid curve) generated by the default prediction model using the realized input values 12 months later.

18

Figure 4: Back testing analysis of Eurozone 12: 12-month PDs using the whole-sample stress testing regressions

The median values of the stressed 12-month PDs for three groups/portfolios over the sample period are reported (dashed curve). At the end of each month starting from January 2002, we run stress testing by always using regressions (2) and (3) that are estimated using the whole sample. The input values for the default prediction model are simulated (1,000 runs) for individual firms based on equations (2) and (3) by assuming a stress economic scenario that was realized 12 months later. For each simulation, we compute the portfolio-median PD for a target group. We then use the mean of the 1,000 simulation runs as the final median value for the group. Also plotted for comparison is the actual median PDs (solid curve) generated by the default prediction model using the realized input values 12 months later.
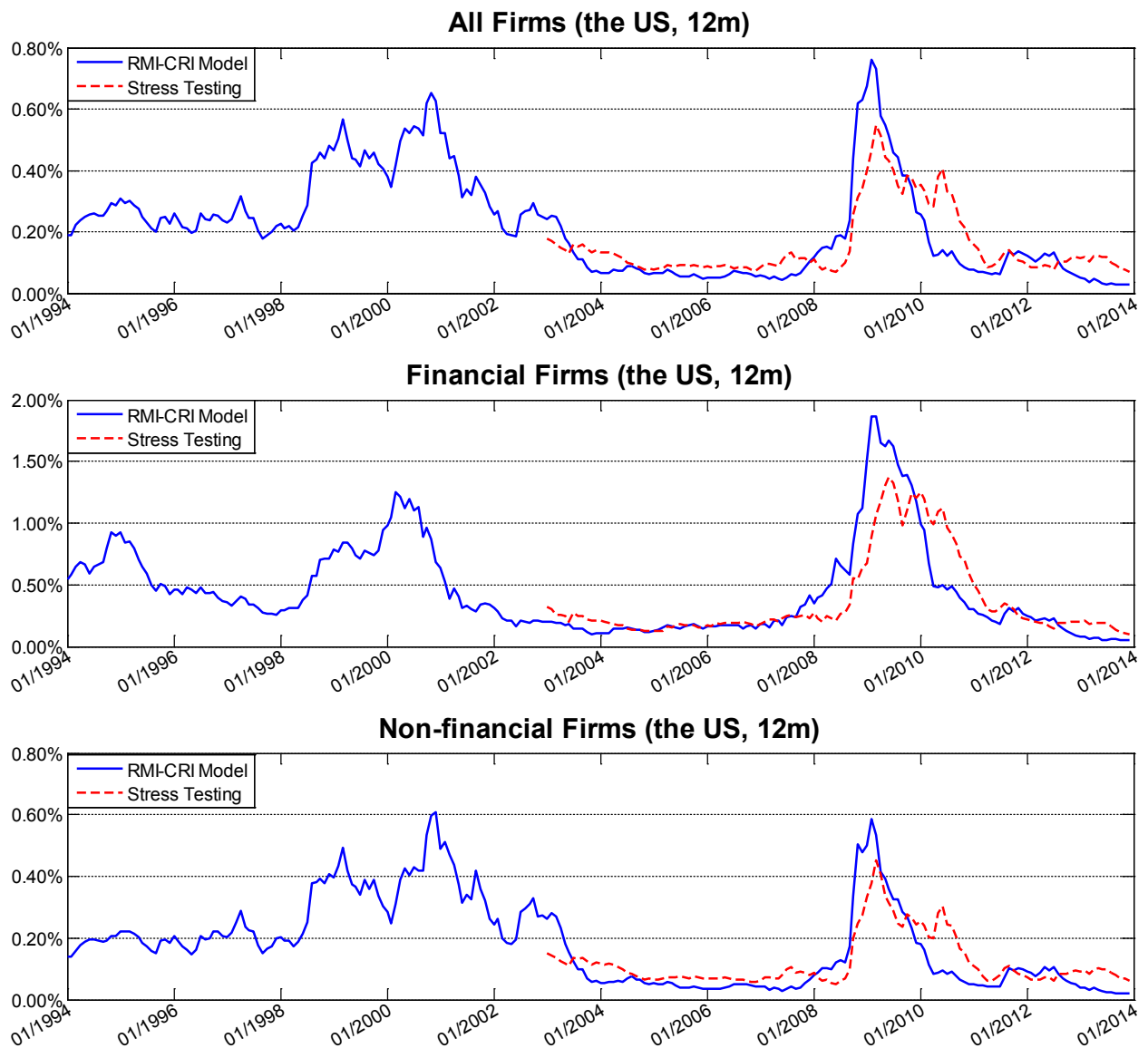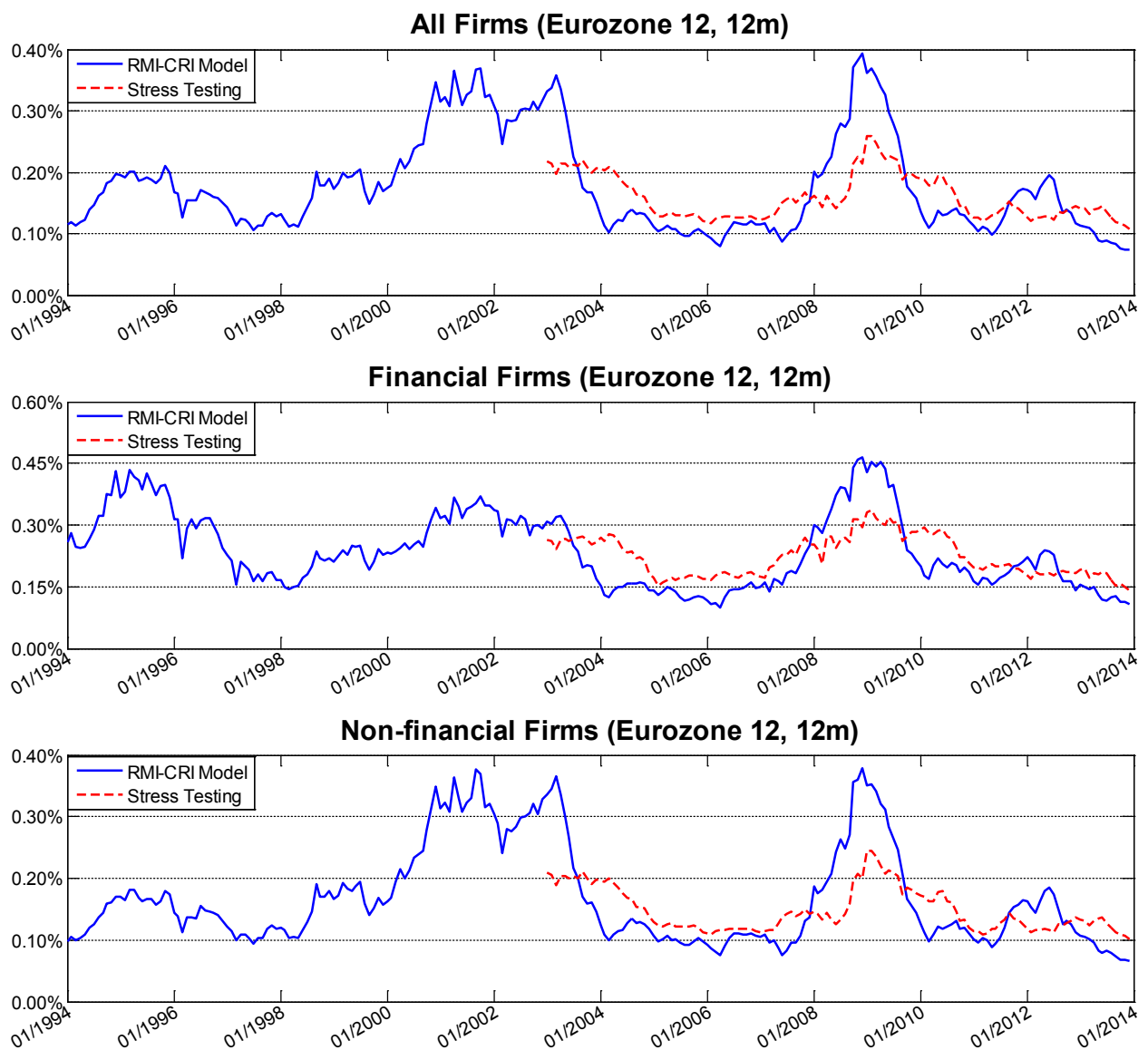
in this group, whereas for the US, it is assumed to be US own real GDP growth rate. In the case of Eurozone 12, the driving economies are assumed to be its two key members – France and Germany. To construct the stress-recovery scenarios, we assume that the annual real GDP growth rate of each driving economy falls concurrently λ standard deviations (computed with historical annual real GDP growth rates) below its baseline projection for the first year, and then returns towards the baseline over next five years according to the following process:

$$Z_s = \theta \cdot Z_{s-1} + (1 - \theta) \cdot B_s. \tag{4}$$

where B_s represents the baseline value of the annual GDP growth rate in year s. In our study, we set ($\lambda = 2, \theta = 0.3$) for the V-shaped recovery, and ($\lambda = 1, \theta = 0.9$) for the protracted stress recovery.

The other stress variables of each economy are expected to respond to the presumed driving GDP paths. A first-order structural vector autoregressive (VAR) model is employed to ensure consistency across all stress variables under the presumed stress scenario. In a nutshell, we let all other stress variables to contemporaneously depend on the real GDP growth rates of the driving economies but not the other way around. This specification ensures that the structural VAR model is identified, and it is natural because the GDP based stress scenario generation is rightly reflected in the specification. In terms of the autoregressive structure, we allow for a variable's own lagged value to enter but all cross autocorrelations are disabled. The technical details on consistent scenario generation are given in Appendix B. Furthermore, when the stress testing involves more than one economy, contemporaneous correlations among stress variables within an economy are allowed. Across economies, however, all stress variables are not directly correlated, but can be correlated indirectly through their links to the driving GDP growth rates. This assumption is made to simplify the model so that fewer parameters need to be estimated.

In practice, one needs baseline projections of real GDP growth rates over future periods in order to implement the IMF-style stress recovery analysis. Policy makers typically have access to baseline projections by some macroeconometric model. In the following analyses, we rely on the reduced form of the aforementioned first-order structural VAR model to generate baseline projections. Naturally, they can be replaced with those projections available to policy makers. We first estimate the first-order VAR model of the real GDP growth rate using quarterly data. Then, quarterly predictions of real GDP growth rates are made for six years, and the other two stress variables corresponding to the real GDP path of the baseline projection are generated by the structural VAR described in Appendix B.

With these simulated baseline projections, the corresponding IMF-style V-shaped and protracted real GDP growth rate paths can be further produced. Note that the IMF-style stress recovery scenario described above specifies six future GDP values annually. To match with the quarterly frequency of the stress variables used in the first-order structural VAR model, we fill in the quarterly values of the presumed paths of real GDP growth rate by setting the quarterly real GDPs to be a quarter of the values obtained by linearly interpolating the annual real GDP values of the six years.
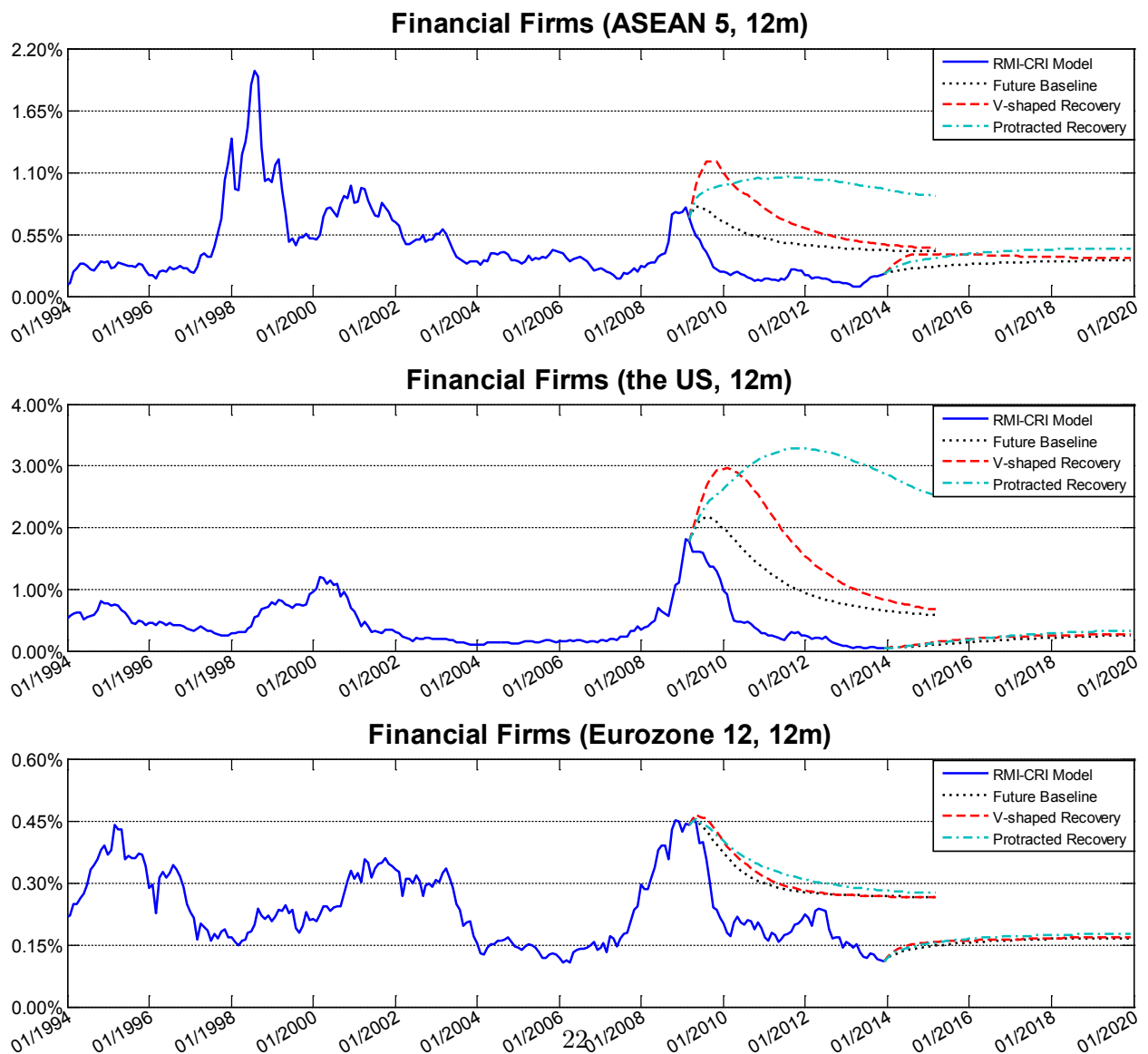
We conduct the V-shaped and protracted stress recovery analyses for two time points: March 2009 and December 2013. In order to appreciate the effect of stress, the outputs corresponding to the baseline projections at these two time points are also produced. For each analysis, we estimate the stress-testing regression equations (2) and (3) and the first-order VAR model in Appendix B with the data accumulated up to the time point of stress testing. We then simulate the stressed macroeconomic environment quarterly over next six years using the first-order structural VAR model to make sure that the other stress variables, although randomly generated, are consistent with the prescribed stress path for the driving real GDP growth rate. Since stress testing is performed monthly, monthly values along the six-year future paths for each of the three stress variables are needed for implementing the stress-testing regressions (2) and (3). Thus, we convert all quarterly values to monthly by linear interpolation. Corresponding to each simulated macroeconomic environment, we further simulate the input variables of the default prediction model monthly over next six years according to the estimated equations (2) and (3). Note that we had earlier described how mixing frequency of quarterly and monthly can be dealt with in these stress-testing regressions. Finally, we adjust an individual firm's PD relative to its industry average as described earlier in the back testing section, and compute the median PD of the target group of firms for each month over six years. We run the simulation 50,000 times, and then calculate the mean values of these medians over six years.

The results for ASEAN 5, the US and Eurozone countries are plotted in Figure 5. The plots are the mean values of the group median 12-month PD paths for the financial sector in March 2009 and December 2013 under the V-shaped and protracted stress recovery scenarios, respectively. At the height of the global financial crisis, i.e., March 2009, the US stressed curves (the middle graph) reveals that credit risk would have kicked up substantially from an already bad situation were the US economy to face a V-shaped or protracted stress recovery from that point onwards. The figures for ASEAN 5 show that stress would have pushed up their relatively high credit risk level another notch although not as pronounced as those for the US. In the case of Eurozone 12, the effect is driven by the baseline projection that is worse than what had subsequently transpired. The difference between the V-shaped (or protracted) recovery outcome vis-a-vis the baseline projection is fairly minor. The result for the US vis-a-vis others reflects much worse solvency and liquidity positions of US financial firms at that time. The V-shaped (or protracted) stress recovery analysis for all three regions reveals very intuitive patterns where the V-shaped curve (dashed) responds to a higher negative initial shock of two standard deviations (or one standard deviation) and then recovers quicker (slower).

It is interesting to note the curves corresponding to the baseline projections of the real GDP growth rate for all three groups. The first-order VAR model used to generate the baseline projections would suggest a slower recovery in credit risk for these countries as compared to what had actually transpired in reality (dotted curve vs. solid curve). This result likely reflects the massive policy interventions, both fiscal and monetary, which brought the GDP recovery at a speed faster than the baseline projections generated by the first-order VAR model. When the V-shaped and protracted stress recovery analysis is conducted at the end of our sample period (December 2013), the conclusion turns out to be quite different. Four years after the global financial crisis, the world

Figure 5: An IMF-style stress recovery analysis of ASEAN 5, the US and Eurozone 12: 12-month PDs

The six-year time series of the median values of the stressed 12-month PDs for the financial sector under the baseline projections (dotted curve), the V-shaped recovery (dashed curve) and the protracted recovery (dash-dotted curve) are reported for two time points – March 2009 and December 2013. The stress-testing regressions (2) and (3) are estimated with data up to the two respective time points. The stress scenario in each analysis is defined by the real GDP growth rate of one or several selected countries, and the other stress variables are made to move in a consistent but random manner. For each simulated stress macro-environment, we further simulate the input values of the default prediction model based on equations (2) and (3) and compute the median PDs (across firms) for the next 72 months. We run 50,000 simulations to calculate the mean values of the median PDs.

Financial Firms (ASEAN 5, 12m)

Financial Firms (the US, 12m)

Financial Firms (Eurozone 12, 12m)

economy has been reasonably stabilized and financial firms have also managed to improve their balance sheets and liquidity postures. A real GDP stress of a magnitude of two standard deviations could hardly move the credit risk profiles for these economies.

5   Conclusions

We propose a readily implementable credit stress testing method that can translate adverse macroeconomic scenarios into different distributions of corporate defaults for an economy and/or any other target portfolio of interest. Our proposed method is a combination of a bottom-up corporate default prediction model, a set of stress-testing regressions which link the macroeconomic stress scenarios to the input variables of the default prediction risk model, and a structural VAR model to generate consistent stress scenarios over multiple stress variables. The default prediction model employed by us is the forward intensity model of Duan, *et al* (2012), which can generate term structures of PDs for individual firms and portfolios. Parameters defining the stress testing equations are estimated by regressing the industry averages of the input variables to the default prediction model on the macroeconomic variables defining the stress scenario. We have also devised a novel way of estimating these stress-testing regressions with mixed-frequency data. The same stress testing equations are then used to simulate future input values under a presumed stress scenario, which in turn lead to PDs of individual firms and aggregate PD for an economy or a target portfolio.

We check performance on a sample consisting of all listed firms in the ASEAN 5 countries. We compare the predicted aggregate PDs by presuming the knowledge of the macroeconomic environment one year ahead with the actual PDs that became available one year later. The results suggest that the proposed stress testing method works well for the ASEAN 5 countries. Additional analysis based on the US and Eurozone 12 data reveals a limit to our stress testing method, which requires a long time series of data including at least a full business cycle with a severe credit crisis. Our out-of-sample performance on the US and Eurozone 12 data for the 2008-09 global financial crisis is therefore predictably unsatisfactory, because during our sample period and prior to the 2008-09 global financial crisis there was no major credit crisis for these economies. However, the in-sample performance, for which the data during the 2008-09 financial crisis are included in estimating the stress-testing regressions, shows adequate performance. This result suggests that our stress testing method is likely to work for the US and Eurozone in future stress testing applications, because the 2008-09 global financial crisis has in a way trained the stress-testing regressions.

Conducting the IMF-style V-shaped and protracted stress recovery analysis leads to some interesting findings. Particularly revealing is the further elevated credit risk for the US financial sector were the US economy to experience a V-shaped or protracted stress recovery scenario at the height of the 2008-09 financial crisis. This analysis in effect demonstrates that our proposed stress testing method coupled with the Credit Research Initiative platform maintained by the Risk Management Institute of National University of Singapore is ready for policy/regulatory applications.

Appendices

A: Treating mixed-frequency data in stress-testing regressions

The stress-testing regressions (2) and (3) will be conducted on the monthly frequency, but GDP and unemployment rate are on a quarterly basis. Thus, the monthly series on GDP growth rate (or change in unemployment rate) exhibits a choppy pattern with the two-thirds of them being zeros. Using such a series of two zeros followed by a big move in the stress-testing regression will seriously bias the parameter estimates. Therefore, we smooth the monthly series by replacing the zeros within a quarter with the linearly interpolated values. But this simple data smoothing treatment is not sufficient to mitigate the bias in the regression estimation because regressors are too smooth within a quarter. Here, we devise a way to further mitigate the bias in estimating the stress-testing regression with our mixed-frequency data.

The idea is to deduce a time-aggregated form for equations (2) and (3) and then use them for estimation. We use equation (2) to illustrate the method. To simplify notation, superscript X and subscript m are dropped. Equation (2) can be expressed in a vector form:

$$\begin{bmatrix} X_t \\ X_{t-1} \end{bmatrix} = \begin{bmatrix} \beta_0 \\ 0 \end{bmatrix} + \sum_{k=1}^{n} \beta_k \begin{bmatrix} Z_{k,t} \\ 0 \end{bmatrix} + \boldsymbol{A} \begin{bmatrix} X_{t-1} \\ X_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \end{bmatrix} \quad \text{with } \boldsymbol{A} = \begin{bmatrix} \gamma_1 + 1 & \gamma_2 \\ 1 & 0 \end{bmatrix}. \tag{5}$$

Let $(\boldsymbol{A}^p)_{i,j}$ denote the (i,j)-th entry of the p-th power of $\boldsymbol{A}$ and $\boldsymbol{A}^0$ be an identity matrix. For any $l \geq 1$, a direct calculation gives rise to

$$X_t = \beta_0 \sum_{p=0}^{l-1} (\boldsymbol{A}^p)_{1,1} + \sum_{k=1}^{n} \sum_{p=0}^{l-1} \beta_k (\boldsymbol{A}^p)_{1,1} Z_{k,t-p} + (\boldsymbol{A}^l)_{1,1} X_{t-l} + (\boldsymbol{A}^l)_{1,2} X_{t-l-1} + \sum_{p=0}^{l-1} (\boldsymbol{A}^p)_{1,1} \varepsilon_{t-p}. \tag{6}$$

As far as parameter estimation is concerned, equation (6) has advantage over equation (2) because the response variable, X_t, is related to each stress variable in its aggregated value over l months. In our later implementation, l is set to 12, i.e., over one year. With $l = 12$, parameter estimation using equation (6) becomes much less sensitive to how the quarterly data are converted into monthly data.

Assume that the disturbance $\{\varepsilon_t\}$ is a Gaussian white noise process, i.e., $\varepsilon_t \sim i.i.d.\ N(0, \sigma^2)$. Let $\boldsymbol{\theta} := (\beta_0, \beta_1, \cdots, \beta_n, \gamma_1, \gamma_2)$. We want to estimate the unknown parameter $(\boldsymbol{\theta}, \sigma)$ with data $\{x_t : t = 1, \cdots, T\}$ and $\{z_{k,t} : k = 1, \cdots, n; t = 3, \cdots, T\}$. Given any $l \geq 1$, for any $t = l+2, \cdots, T$, define the l-period predicted value at time t as

$$\hat{x}_t^{(l)}(\boldsymbol{\theta}) := \beta_0 \sum_{p=0}^{l-1} (\boldsymbol{A}^p)_{1,1} + \sum_{k=1}^{n} \sum_{p=0}^{l-1} \beta_k (\boldsymbol{A}^p)_{1,1} z_{k,t-p} + (\boldsymbol{A}^l)_{1,1} x_{t-l} + (\boldsymbol{A}^l)_{1,2} x_{t-l-1}. \tag{7}$$

Let $\boldsymbol{x} := (x_{l+2}, \cdots, x_T)'$, $\hat{\boldsymbol{x}}^{(l)}(\boldsymbol{\theta}) := \left(\hat{x}_{l+2}^{(l)}(\boldsymbol{\theta}), \cdots, \hat{x}_T^{(l)}(\boldsymbol{\theta}) \right)'$ and $\boldsymbol{\varepsilon} := (\varepsilon_3, \cdots, \varepsilon_{l+1}, \varepsilon_{l+2}, \cdots, \varepsilon_T)'$. By equation (6), we can express the residual vector as

$$\boldsymbol{x} - \hat{\boldsymbol{x}}^{(l)}(\boldsymbol{\theta}) = \boldsymbol{C}^{(l)}(\boldsymbol{\theta})\, \boldsymbol{\varepsilon},$$

where $\boldsymbol{C}^{(l)}(\boldsymbol{\theta})$ is a $(T - l - 1) \times (T - 2)$ Toeplitz matrix as follows:

$$\boldsymbol{C}^{(l)}(\boldsymbol{\theta}) := \begin{bmatrix} (\boldsymbol{A}^{l-1})_{1,1} & (\boldsymbol{A}^{l-2})_{1,1} & \cdots & (\boldsymbol{A}^0)_{1,1} & 0 & \cdots & 0 & 0 \\ 0 & (\boldsymbol{A}^{l-1})_{1,1} & (\boldsymbol{A}^{l-2})_{1,1} & \cdots & (\boldsymbol{A}^0)_{1,1} & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & (\boldsymbol{A}^{l-1})_{1,1} & (\boldsymbol{A}^{l-2})_{1,1} & \cdots & (\boldsymbol{A}^0)_{1,1} & 0 \\ 0 & 0 & \cdots & 0 & (\boldsymbol{A}^{l-1})_{1,1} & (\boldsymbol{A}^{l-2})_{1,1} & \cdots & (\boldsymbol{A}^0)_{1,1} \end{bmatrix}.$$

It follows that $\boldsymbol{x} - \hat{\boldsymbol{x}}^{(l)}(\boldsymbol{\theta}) \sim N\big(\boldsymbol{0}, \sigma^2 \boldsymbol{\Sigma}^{(l)}(\boldsymbol{\theta})\big)$ with $\boldsymbol{\Sigma}^{(l)}(\boldsymbol{\theta}) := \boldsymbol{C}^{(l)}(\boldsymbol{\theta})\boldsymbol{C}^{(l)}(\boldsymbol{\theta})'$. Therefore, the log-likelihood function (conditioned on the first two observations) takes the form:

$$L(\boldsymbol{\theta}, \sigma) = -\frac{T - l - 1}{2} \log(2\pi) - \frac{1}{2} \log\big(|\sigma^2 \boldsymbol{\Sigma}^{(l)}(\boldsymbol{\theta})|\big) - \frac{1}{2}\big(\boldsymbol{x} - \hat{\boldsymbol{x}}^{(l)}(\boldsymbol{\theta})\big)'\big(\sigma^2 \boldsymbol{\Sigma}^{(l)}(\boldsymbol{\theta})\big)^{-1}\big(\boldsymbol{x} - \hat{\boldsymbol{x}}^{(l)}(\boldsymbol{\theta})\big). \quad (8)$$

Parameter estimates $(\boldsymbol{\theta}^*(l), \sigma^*(l))$ can then be obtained by maximizing the above log-likelihood function.

We use R^2 to measure the goodness of fit of the estimated model at its maximum likelihood estimate $\boldsymbol{\theta}^*$. This statistic can be expressed as the ratio of the variance of model's prediction (explained variance) to the sample variance of the dependent variable (total variance), but it must reflect the nature of time series dependency induced by time aggregation. For an s-period prediction performance ($s \geq 1$ and can be different from l), two vectors of s-period difference are defined:

$$\Delta_s \boldsymbol{x} := (x_{s+2} - x_2, \cdots, x_T - x_{T-s})' \quad \text{and} \quad \Delta_s \hat{\boldsymbol{x}}^{(s)}(\boldsymbol{\theta}^*) := \big(\hat{x}_{s+2}^{(s)}(\boldsymbol{\theta}^*) - x_2, \cdots, \hat{x}_T^{(s)}(\boldsymbol{\theta}^*) - x_{T-s}\big)' \quad (9)$$

where $\hat{x}_t^{(s)}$ is the s-period prediction at time t defined as in equation (7). Then, the s-period R^2 can be expressed as

$$R^2 := 1 - \frac{\big(\Delta_s \boldsymbol{x} - \Delta_s \hat{\boldsymbol{x}}^{(s)}(\boldsymbol{\theta}^*)\big)' \big(\boldsymbol{\Sigma}^{(s)}(\boldsymbol{\theta}^*)\big)^{-1} \big(\Delta_s \boldsymbol{x} - \Delta_s \hat{\boldsymbol{x}}^{(s)}(\boldsymbol{\theta}^*)\big)}{\big(\Delta_s \boldsymbol{x} - \overline{\Delta_s \boldsymbol{x}}\big)' \big(\boldsymbol{\Sigma}^{(s)}(\boldsymbol{\theta}^*)\big)^{-1} \big(\Delta_s \boldsymbol{x} - \overline{\Delta_s \boldsymbol{x}}\big)}, \quad (10)$$

where $\overline{\Delta_s \boldsymbol{x}}$ denotes the mean of $\Delta_s \boldsymbol{x}$. Weighting by $\big(\boldsymbol{\Sigma}^{(s)}(\boldsymbol{\theta}^*)\big)^{-1}$ reflects overlapped s-period differences. This R^2 attains its maximum value of 1 for a model of perfect fit. Negative values may occur when s differs from l. Reported in Tables 3-5 are the R^2 for the 12-period (month) prediction, i.e., $s = 12$.

B: Generating consistent stress paths by a structural VAR

We use three stress variables ($Z_{1,t}$: real GDP growth rate, $Z_{2,t}$: change in unemployment rate, and $Z_{3,t}$: inflation rate) to show how consistent paths can be generated with a first-order structural

VAR model. Consider only one economy whose real GDP growth rate is the driving stress variable. Our first-order structural VAR takes the form:

$$\begin{bmatrix} 1 & 0 & 0 \\ \beta_{21} & 1 & \beta_{23} \\ \beta_{31} & \beta_{32} & 1 \end{bmatrix} \begin{bmatrix} Z_{1,t} \\ Z_{2,t} \\ Z_{3,t} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix} + \begin{bmatrix} \rho_1 & 0 & 0 \\ 0 & \rho_2 & 0 \\ 0 & 0 & \rho_3 \end{bmatrix} \begin{bmatrix} Z_{1,t-1} \\ Z_{2,t-1} \\ Z_{3,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix} \tag{11}$$

where $\varepsilon_{1,t}$, $\varepsilon_{2,t}$ and $\varepsilon_{3,t}$ are independent of one another. The above structural VAR is identified because of the restrictions placed on the contemporaneous relations and the coefficient matrix for the lagged term.

The corresponding reduced-form model can be used for estimation and generating the baseline projection of the real GDP growth path; that is,

$$\begin{bmatrix} Z_{1,t} \\ Z_{2,t} \\ Z_{3,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \beta_{21} & 1 & \beta_{23} \\ \beta_{31} & \beta_{32} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ \beta_{21} & 1 & \beta_{23} \\ \beta_{31} & \beta_{32} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \rho_1 & 0 & 0 \\ 0 & \rho_2 & 0 \\ 0 & 0 & \rho_3 \end{bmatrix} \begin{bmatrix} Z_{1,t-1} \\ Z_{2,t-1} \\ Z_{3,t-1} \end{bmatrix}$$
$$+ \begin{bmatrix} 1 & 0 & 0 \\ \beta_{21} & 1 & \beta_{23} \\ \beta_{31} & \beta_{32} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix}. \tag{12}$$

More conveniently, the structural VAR model in (11) can also be simplified into two sub-systems because of its special structure; that is,

$$Z_{1,t} = \mu_1 + \rho_1 Z_{1,t-1} + \varepsilon_{1,t} \tag{13}$$

and

$$\begin{bmatrix} 1 & \beta_{23} \\ \beta_{32} & 1 \end{bmatrix} \begin{bmatrix} Z_{2,t} \\ Z_{3,t} \end{bmatrix} = \begin{bmatrix} \mu_2 \\ \mu_3 \end{bmatrix} - \begin{bmatrix} \beta_{21} Z_{1,t} \\ \beta_{31} Z_{1,t} \end{bmatrix} + \begin{bmatrix} \rho_2 & 0 \\ 0 & \rho_3 \end{bmatrix} \begin{bmatrix} Z_{2,t-1} \\ Z_{3,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix}. \tag{14}$$

The sub-system in (14) governs the other two variables contingent upon $Z_{1,t}$, and its reduced form can be conveniently expressed as

$$\begin{bmatrix} Z_{2,t} \\ Z_{3,t} \end{bmatrix} = \begin{bmatrix} \mu_2' \\ \mu_3' \end{bmatrix} - \begin{bmatrix} \beta_{21}' Z_{1,t} \\ \beta_{31}' Z_{1,t} \end{bmatrix} + \begin{bmatrix} \rho_{11}' & \rho_{12}' \\ \rho_{21}' & \rho_{22}' \end{bmatrix} \begin{bmatrix} Z_{2,t-1} \\ Z_{3,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{2,t}' \\ \varepsilon_{3,t}' \end{bmatrix} \tag{15}$$

where $\varepsilon_{2,t}'$ and $\varepsilon_{3,t}'$ are correlated. Therefore, the two sub-systems in (13) and (14) are nothing but an AR(1) model for $Z_{1,t}$ and a VAR(1) model for $(Z_{2,t}, Z_{3,t})$ with an exogenous variable $Z_{1,t}$. With a presumed stressed real GDP growth rate path, the reduced form in (15) can be used to generate changes in unemployment rate and the inflation rate in a consistent manner.

The system in (11) can be straightforwardly expanded to a higher dimension when the following situations occur individually or simultaneously: (a) If there are m driving economies, then the deduced sub-system in (13) will be an m-dimensional VAR(1) model for m driving GDP growth

paths; (b) If a stress testing economy is not a driving one, then the deduced sub-system in (14) will become three-dimensional, including its real GDP growth rate as the additional element.

To handle multiple stress testing economies, one can actually take care of one economy at a time due to our simplifying assumption that contemporaneous correlations among stress variables within an economy are allowed, but no direct correlations are permitted across economies.

References

[1] Alessandri, Piergiorgio, Prasanna Gai, Sujit Kapadia, Nada Mora, and Claus Puhr, 2009, Towards a framework for quantifying systemic stability, *International Journal of Central Banking* 5, 47-81.

[2] Allen, Linda, and Anthony Saunders, 2004, Incorporating systemic influences into risk measurements: A survey of the literature, *Journal of Financial Services Research* 26, 161-191.

[3] Åsberg Sommar, Per, and Hovick Shahnazarian, 2008, Macroeconomic impact on expected default freqency, Riksbank Working Paper No. 220.

[4] Avesani, Renzo, Kexue Liu, Alin Mirestean, and Jean Salvati, 2006, Review and implementation of credit risk models of the financial sector assessment program (FSAP), IMF Working Paper No. 06/134.

[5] Blaschke, Winfrid, Matthew T. Jones, Giovanni Majnoni, and Maria S. Martinez Peria, 2001, Stress testing of financial systems: An overview of issues, methodologies, and FSAP experiences, IMF Working Paper No. 01/88.

[6] Burrows, Oliver, David Learmonth, and Jack McKeown, 2012, RAMSI: a top down stress testing model, Financial Stability Paper No. 17.

[7] Campbell, John Y., Jens Hilscher, and Jan Szilagyi, 2008, In search of distress risk, *Journal of Finance* 63, 2899-2939.

[8] Castrén, Olli, Trevor Fitzpatrick, and Matthias Sydow, 2009, Assessing portfolio credit risk changes in a sample of eu large and complex banking groups in reaction to macroeconomic shocks, European Central Bank Working Paper Series No. 1002.

[9] Chan-Lau, Jorge A., 2013, Market-based structural top-down stress tests of the banking system, IMF Working Paper No. 13/88.

[10] Chopra, Gautam, 2009, Stress testing financial systems: A macro perspective, Government of India.

[11] Coletti, Don, Rene Lalonde, Miroslav Misina, Dirk Muir, Pierre St-Amant, and David Tessier, 2007, Bank of canada participation in the 2007 FSAP macro stress testing excercise, Bank of Canada.

[12] Duan, Jin-Chuan, Jie Sun, and Tao Wang, 2012, Multiperiod corporate default prediction - a forward intensity approach, *Journal of Econometrics* 170(1), 191-209.

[13] Duffie, D., L. Saita, and K. Wang, 2007, Multi-period corporate default prediction with stochastic covariates, *Journal of Financial Economics* 83, 635-665.

[14] Eklund, Trond, Kai Larsen, and Eivind Berhardsen, 2001, Model for analysing credit risk in the enterprise sector, *Economic Bulletin* 72(3), 99-106.

[15] Ferry, Danielle H., Tony Hughes, and Min Ding, 2012a, Stressed EDFTM credit measures, Modelling Methodology (Moody's Analytics).

[16] Ferry, Danielle H., Tony Hughes, and Min Ding, 2012b, Stressed EDFTM credit measures for Western Europe, Modelling Methodology (Moody's Analytics).

[17] Foglia, Antonella, 2009, Stress testing credit risk: A survey of authorities' approaches, *International Journal of Central Banking* 5, 9-45.

[18] Foglia, Antonella, Roberta Fiori, and simonetta Iannotti, 2009, Beyond macroeconomic risk: The role of contagion in the Italian corporate default correlation, CAREFIN Research Paper No. 12/09.

[19] Ghysels, E., A. Sinko and R. Valkanov, 2007, MIDAS regressions: further results and new directions, *Econometric Reviews* 26(1), 53-90.

[20] Hoggarth, Glenn, Steffen Sorensen, and Lea Zicchino, 2005, Stress tests of UK banks using a VAR approach, Bank of England Working Paper Series No. 282.

[21] IMF, 2012, Marcrofinancial stress testing – Principles and practices, International Monetary Fund.

[22] Jarrow, Robert A., and Stuart M. Turnbull, 2000, The intersection of market and credit risk, *Journal of Banking and Finance* 24, 271-299.

[23] Jobst, Andreas A., Li Lian Ong and Christian Schmieder, 2013, A framework for macropreduential bank solvency stress testing: Application to S-25 and other G-20 country FSAPs, IMF Working Paper No. WP/12/68.

[24] Jones, Matthew T., and Paul Hilbers, 2004, Stress testing financial systems: What to do when the governor calls, IMF Working Paper No. WP/04/127.

[25] Lehmann, Hansjorg and Michael Manz, 2006, The exposure of Swiss banks to macroeconomic shocks – an empirical investigation, Swill National Bank Working Papers 2006-04.

[26] Mencia, Javier, and Gabriel Jimenez, 2007, Modelling the distribution of credit losses with observable and latent factors, *Journal of Empirical Finance* 16, 235-253.

[27] Misina, Miroslav, and David Tessier, 2008, Non-linearities, model uncertainty, and macro stress testing, Bank of Canada Working Paper 2008-30.

[28] Moretti, Marina, Stephanie Stolz, and Mark Swinburne, 2008, Stress testing at the IMF, IMF Working Paper No. 08/206.

[29] Pesaran, M. Hashem, Til Schuermann, Bjorn-Jakob Treutler, and Scott M. Weiner, 2006, Macroeconomic dynamics and credit risk: A global perspective, *Journal of Money, Credit and Banking* 38, 1211.

[30] RMI-CRI Technical Report (Version 2013 Update 2b), 2013, Risk Management Institute, National University of Singapore.

[31] Sorge, Marco, 2004, Stress-testing financial systems: An overview of current methodologies, BIS Working Paper No. 165.

[32] van den End, Jan Willem, Marco Hoeberichts, and Mostafa Tabbae, 2006, Modelling scenario analysis and macro stress testing, Netherlands Central Bank DNB Working Papers.

[33] Virolainen, Kimmo, 2004, Macro stress testing with a macroeconomic credit risk model for Finland, Bank of Finland Discussion Paper No. 18/2004.

[34] Wilson, Thomas, 1997a, Portfolio credit risk I, *Risk* 10(9), 111-117.

[35] Wilson, Thomas, 1997b, Portfolio credit risk II, *Risk* 10(10), 56-61.

[36] Worrell, DeLisle., 2004, Quantitative assessment of the financial sector: An integrated approach, IMF Working Paper No. WP/04/153.