Liquidity and Default

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Abstract

Market liquidity is informative of future corporate defaults but in a nuanced way. A firm's probability of default increases with market illiquidity only when the firm's funding liquidity is tight and/or solvency position is weak. Such relationship persists after controlling for a variety of variables that are known to be related to corporate defaults. The effect of market illiquidity on default probabilities is as large as 638 bps on an annualized basis during the 2007-2009 financial crisis period. Even in a non-crisis period, the impact on default probabilities can be large enough to downgrade a firm's credit rating by a notch. Our conclusion is robust to different modeling approaches and choices of the set of default predictors.

Keywords: market liquidity, funding liquidity, distance-to-default, solvency, forward intensity, logistic regression

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1 Introduction

Liquidity can refer to two different but intertwined concepts: funding liquidity (i.e., ease to obtain funding) and market liquidity (i.e., ease to trade without unduely affecting prices). The interaction of these two types of liquidity has been, for example, articulated and modeled in Brunnermeier and Pedersen (2009) to show liquidity spirals. A firm's funding liquidity is its ability to pay short-term obligations by its cash on hand and/or ability to borrow in short notice, which may, for example, be measured empirically by cash over total assets or working capital over current liabilities. Market liquidity, on the other hand, refers to the ease with which assets can be disposed of quickly at a reasonable price, but opinions are divided on how best to measure this liquidity due to its complexity; see, for example, Sarr and Lybet (2002).

The fact that funding liquidity is important in default prediction is widely recognized (see Beaver (1966), Altman (1968), Ohlson (1980), Zmijewski (1984), etc.), but the understanding remains limited regarding the importance of market liquidity in predicting defaults. The objective of this paper is to investigate how market liquidity (illiquidity) affects corporate defaults and to determine whether it can help default prediction. In this paper, we focus on a market illiquidity measure proposed in Hu, *et al* (2013), which is based on the noise in the US government bond yield curve.

Repaying corporate debt requires financial resources, and it seems reasonable to conjecture that firms facing a severe discount in selling their assets in an illiquid market should be more prone to default. For a firm that employs some form of leverage, its ability to repay debt depends on its funding liquidity as well as market liquidity. Funding liquidity provides internally available cash, whereas market liquidity affects its external funding ability at the time of needs. In a more liquid market, firms gain easier access to funds at a lower cost through raising capital or selling assets. It is so either because there are more capital available in the markets or because substitutability among various assets becomes higher. The external funding ability becomes crucial in terms of default only if the firm's internal funding is tight and/or its solvency condition is weak. In essence, when a firm needs outside funds to repay debt, market liquidity will become a critical factor in determining its survivability. Its likelihood of default naturally increases (decreases) under an illiquid (liquid) market condition.

Our study uses a sample that comprises monthly data on US public firms, both financial and non-financial, from January 1990 to December 2012. Inclusion of financial firms in default prediction is made possible by suitably measuring distance-to-default for financial firms as in Duan, *et al* (2012). We establish in this paper that corporate default probabilities are strongly influenced by market illiquidity, but its impact is limited to firms with low funding liquidity and/or in a poor solvency position. Only when a firm's funding liquidity is weak and/or solvency is challenged, its default probabilities over shorter horizons will become responsive to the market liquidity condition. Our conclusion is reached by introducing a market liquidity (illiquidity) measure into the forward-intensity corporate default prediction model of Duan, *et al* (2012) which characterizes firms cross-sectionally by their individual attributes and the state of economy time series wise by macroeconomic risk factors. Our conclusion on market liquidity (illiquidity) is found to be robust by controlling for many variables that are known to be related to corporate default. Robustness of our conclusion is further ascertained by employing a standard but less versatile default prediction model such as that of Shumway (2001), and strictly applying on a sample of non-financial firms per a typical practice in the literature.

Our findings on market liquidity are consistent with those reported in the corporate bond literature. It is widely known that market illiquidity can cause higher corporate bond yields such that credit spread can be decomposed into liquidity premium and default premium components (e.g., Longstaff, *et al* 2005). Ericsson and Renault (2006) developed a structural bond valuation model to show that illiquidity and default component of bond yield spreads are positively correlated. This is because renegotiation in financial distress is influenced by the market illiquidity for distressed debt. Their empirical results show that both aggregate illiquidity in the Treasury market and individual issue's illiquidity are positively related to yield spread and the impact is larger for bonds with a lower credit rating. Conceptually, market illiquidity can influence the yield spread of a bond through two channels – (1) directly affects bond investors' willingness to hold this bond and thus a higher yield spread, and (2) creates difficulties in raising external funds to stave off default which in turn indirectly pushes up yield spread through a higher default premium. The former is rightly the liquidity premium, but the latter is actually the default premium in disguise. In other words, without properly factoring market liquidity into default probability estimates, one risks overstating liquidity premiums simply due to shutting off the indirect channel. Our paper can thus be viewed as an effort to obtain a better default probability estimate for the obligor, which in turn leads to a more meaningful instrument-specific liquidity risk premium.

He and Xiong (2012) developed a theoretical model to study the interaction between debt market liquidity and credit risk. Their model shows that when debt market liquidity goes down, equity holders face increasing rollover losses in issuing new bond to replace maturing bond. This causes conflict of interest since equity holders suffer losses while debt holders get paid in full. The conflict may lead equity holders to choose an earlier default. He and Milbradt (2013) went one step further by endogenizing market liquidity to feature liquidity-driven-default and default-driven-liquidity components in addition to pure liquidity and default components in credit spread. Our reduced-form econometric model in essence explores this interesting aspect of liquidity-driven-default.

Our paper adds to the large literature on corporate default/bankruptcy prediction. The exploration of models and variables that can help predict bankruptcy originated from the ratio analysis of Beaver (1966) and the multiple discriminant analysis (MDA) of Altman (1968), and both provided evidence that accounting variables, such as current ratio, net income, sales, market to book ratio of equity, etc., are helpful in predicting firm failures. Though MDA was the dominant method back then, its assumptions are too restrictive to hold in real data (see Ohlson (1980)). Subsequently, binary response models became popular in the default prediction literature, such as the logit model in Martin (1977), Ohlson (1980), the probit model in Zmijewski (1984), the hazard rate model (or dynamic logit model) in Shumway (2001), Chava and Jarrow (2004) and Campbell, *et al* (2008).

Recently, default prediction models based on doubly stochastic Poisson processes while factoring in other forms of corporate exit have emerged in the literature; for example, Duffie, *et al* (2007), Duffie, *et al* (2009) and Duan, *et al* (2012). Other than accounting variables, Shumway (2001) found that market variables such as the excess stock return and the volatility of stock returns are also significant in predicting bankruptcy. Chava and Jarrow (2004) showed that adding industry effects can significantly improve default/bankruptcy prediction. Our paper adds to the literature by showing that market liquidity is both statistically and economically significant in default/bankruptcy prediction.

The remainder of this paper proceeds as follows. Section 2 describes the forward-intensity default prediction model used in testing our hypotheses. In addition, the input variables for the default prediction model and their data sources are described. Section 3 presents and discusses empirical results including economic significance of including market liquidity. Section 4 conducts a robustness check using a dynamic logit model of Shumway (2001) and three different sets of default predictors, and Section 5 concludes.

2 Methodology

2.1 Default Prediction with a Censoring Control for Other Corporate Exits

Intensity-based approaches to corporate default modeling have appeared in the literature in recent years. Default is modeled as an unpredictable Poisson event with stochastic intensities whereas exit for reasons other than default (e.g., merger and acquisition) is treated as another independent Poisson event with different stochastic intensities. The two intensities can be correlated, but the event of default and that of other exits are independent, however. To our knowledge, Duffie, *et al* (2007) is the first to propose such a doubly stochastic Poisson intensity model to describe default event and other forms of corporate exit, where the two types of intensities are specified as two universal functions of some stochastic covariates (firm-specific attributes and macroeconomic factors). Thus, default correlations occur in their model through the correlations in the stochastic covariates. Their model is capable of producing probabilities of default (PDs) over multiple horizons. In order to do so, however, one must also specify an auxiliary model to govern the dynamic evolution of the stochastic covariates, which turns out to be a quite challenging task due to the high dimensionality of this auxiliary model; for example, a sample of 5,000 corporates with four firm-specific attributes plus two macroeconomic risk factors will amount to an auxiliary system with a dimension of 20,002.

Duan, *et al* (2012) proposed a forward-intensity model as an alternative to the spot-intensity approach of Duffie, *et al* (2007). Only using data available at the time of performing prediction, the forward-intensity model can produce the term structure of PDs without needing to specify the auxiliary system for the stochastic covariates. The forward-intensity model relies on a set of forward-intensity functions indexed by the forward starting time, and it is a practical econometric tool because through its unique decomposability, the parameters for different forward-intensity functions can actually be independently estimated and subsequently assembled together in applications. As evidence of practicality, this forward-intensity model of Duan, *et al* (2012) has been implemented and operated by the Risk Management Institute (RMI) of the National University of Singapore since July 2010 under its Credit Research Initiative (CRI). The RMI-CRI currently uses this model to produce daily updated PDs, ranging from one month to five years, for about 35,000 currently active public firms and 30,000 delisted firms in 106 economies in the world, and the PDs are freely accessible.[1] In this paper, we will incorporate market liquidity into the forward-intensity model of Duan, *et al* (2012) to study the impacts of market liquidity.

The forward intensity function can be specified as an exponential of some linear combination of input variables (i.e., firm-specific attributes and macroeconomic risk factors) or in another functional form as long as it is a positive function. The specification in Duan, *et al* (2012) for firm i's forward default intensity at t, with 12 input variables $X_{it} = (x_{it,1}, x_{it,2}, \cdots, x_{it,12})$ and a forward starting time τ, is in the following form:

$$f_{it}(\tau) = \exp\left[\beta_0(\tau) + \beta_1(\tau)x_{it,1} + \beta_1(\tau)x_{it,1} + \cdots + \beta_{12}(\tau)x_{it,12}\right] \tag{1}$$

where $\beta(\tau) = [\beta_0(\tau), \beta_1(\tau), \cdots, \beta_{12}(\tau)]$ is the coefficient vector. When $\tau = 0$, the forward-intensity function becomes a spot intensity function like one in Duffie, *et al* (2007). The forward-intensity function for other exits, denoted by $h_{it}(\tau)$, is similarly specified but with a different coefficient vector. Although $f_{it}(\tau)$ and $h_{it}(\tau)$ are stochastic over time, they are known at time t so that forward or cumulative PDs over different horizons from time t onwards can be computed easily. For example, the PD over two years can be calculated as $\int_0^2 e^{-\int_0^\tau [f_{it}(s)+h_{it}(s)]ds} f_{it}(\tau)d\tau$. Note that survival is governed by a combined forward intensity of $[f_{it}(s) + h_{it}(s)]$ to reflect the fact that a firm can exit for default or other reasons. Interestingly, most of the default/bankruptcy prediction literature has ignored the censoring bias related to other exits, which according to Table 1 of Duan, *et al* (2012), the likelihood for a typical US public firm to exit for reasons other than default can easily be ten times the rate of default.

The 12 covariates in Duan, *et al* (2012) include two common macro-financial variables and several firm-specific attributes. The common variables are the trailing one-year simple return of the S&P500 index and the 3-month US Treasury yield. There are six firm-specific variables reflecting funding liquidity, solvency, profitability, relative size, market misvaluation/future growth opportunities, and idiosyncratic volatility. Funding liquidity is measured as the ratio of cash and short-term investments to total assets; profitability as the ratio of net income to total assets; solvency as reflected in a firm's distance-to-default (DTD); relative size as the logarithm of the ratio of market capitalization to the median market capitalization; idiosyncratic volatility is Shumway's (2001) measure of the idiosyncratic component of a firm's equity return risk. Among the six firm-specific attributes, four (i.e., funding liquidity, solvency, profitability and relative size) undergo a simple transformation to represent level and trend. Level is computed as the one-year moving average of any one of the measures corresponding to the four attributes, and trend is computed as its current value minus the one-year moving average. Level is used to differentiate across firms whereas trend is supposed to be indicative of a firm's future direction for a particular variable.

[1] Readers are referred to the RMI-CRI website (`http://rmicri.org/`) for further details.

With the transformation, there are all together ten firm-specific attributes (i.e., two without the level-trend transformation plus eight due to the transformation of four variables).

To examine the role of market liquidity in default prediction, we add it to the forward-intensity function as the 13th variable:

$$f^*_{it}(\tau) = \exp\left[\beta_0(\tau) + \beta_1(\tau)x_{it,1} + \beta_1(\tau)x_{it,1} + \cdots + \beta_{12}(\tau)x_{it,12} + \beta_{13}(\tau)x_{it,13}\right] \tag{2}$$

While $(x_{it,1}, x_{it,2}, \cdots, x_{it,12})$ is the same set of covariates as in Duan, *et al* (2012), $x_{it,13}$ is the place we introduce the market liquidity (illiquidity) variable, which will be specified in three different ways to correspond to our three hypotheses studied later. The forward-intensity function for other corporate exits, $h_{it}(\tau)$, also follows the same set of 13 covariates.

2.2 Hypotheses on the Role of Market Liquidity (Illiquidity)

Research on market liquidity has grown rapidly over the past decade influencing our understanding of asset pricing and corporate finance. With the wide application of measures such as bid-ask spread, there is often the assumption that "liquidity proxies capture the transaction costs of market participants (Goyenko, *et al* 2009). However, transaction cost of trading is only one of the many dimensions that liquidity captures. Sarr and Lybet (2002) discussed five characteristics a liquid market exhibit: (1) Tightness, which means low transaction cost and implicit costs; (2) Immediacy, or the speed with which orders can be executed; (3) Depth, or the existence of abundant orders; (4) Breath, meaning that orders are large in volume with minimal impact on prices; (5) Resiliency, meaning quick correction of price deviation from fundamentals. The problem is that most measures cannot fully account for all these dimensions. Bid-ask spread mainly captures the tightness aspect, whereas Amihud's illiquidity (Amihud (2002)) focuses on depth and breath, and has been shown to perform well in measuring price impact (Goyenko, *et al* 2009). The liquidity measure by Pastor and Stambaugh (2003), on the other hand, relies on the resiliency aspect since it is based on the idea that an asset with lower liquidity corresponds to bigger price impact, therefore stronger volume-related return reversals.

The market illiquidity measure adopted for our analysis is that of Hu, *et al* (2013), and its time t value is denoted by MIL_t. The bond noise measure of Hu, *et al* (2013) is a more encompassing and direct measure of market-wide illiquidity. It utilizes the idea that the amount of arbitrage capital supplies market-wide trading liquidity, which can eliminate arbitrages in the market, resulting in lower price deviation, i.e., lower noises. Rather than trying to capture each of the dimensions in liquidity, the noise measure is able to proxy market illiquidity as reflected in the resulting trading prices, because higher market liquidity results in smaller noises. Although the noise measure is constructed from the US Treasury bond market, it contains information about liquidity needs for the broader financial market, because the US Treasury instruments based repos and reverse repos constitute a key way of short-term financing/investment. It is not too far-fetched to say that the bond noise measure reflects the degree of liquidity condition in the aggregate financial market. It thus serves our purpose well because our study focuses on the role of market-wide illiquidity, instead of individual asset's market illiquidity.[2]

With a suitable market illiquidity measure in place, we test three specific hypotheses to determine how the likelihood of corporate default is impacted by it.

Hypothesis 1: Market illiquidity raises PD for firms with tight funding liquidity

The impact of market illiquidity may be subjected to differences in individual funding liquidity, and the hypothesis is examined through the variable, $x_{it,13}$, which is firm-specific due to varying funding liquidity positions of individual firms:

$$x^{(1)}_{it,13}(k) = MIL_t \times 1_{\{FL_{it} \in k \text{ deciles}\}},\ k = 1, 2, \cdots, 10 \tag{3}$$

Using $x^{(1)}_{it,13}(k)$ in equation (2) will be referred to as **Model 1**, which serves as the basis for testing Hypothesis 1. The funding liquidity measure (FL_{it}) follows the same definition as in Duan, *et al* (2012), i.e., the ratio of cash and short-term investments to total assets. The indicator function, $1_{\{FL_{it} \in k \text{ deciles}\}}$, gives rise to a value of 1 if FL_{it} is in the k-th percentile or lower, and it returns zero if otherwise. The percentile is calculated using all public firms at time t, so that tightness of individual funding liquidity is relative and not affected by time-varying pattern in the aggregate level of funding liquidity. A positive and significant coefficient on $x^{(1)}_{it,13}(k)$ indicates that the forward-default intensity is increasing with market illiquidity (i.e., more likely to default). Our conjecture is that it will be so when k is low (i.e., weak funding liquidity).

One might consider defining $x^{(1)}_{it,13}(k)$ strictly based on a funding liquidity decile as opposed using the cumulative measure from the lowest decile up. There are two reasons for using a cumulative measure. The first has to do with an identification issue. Take the top decile as an example, the firms in this category of highest funding liquidity are highly unlikely to default, and thus provide no and few data points to meaningfully estimate the coefficient. Second, note that $x^{(1)}_{it,13}(10) = MIL_t$, which means that we are progressively enlarging the segment of firms in terms of funding liquidity and subjecting them to market illiquidity until eventually covers the entire population.

Hypothesis 2: Market illiquidity raises PD for firms with weak solvency

Similarly, the second test examines the role of market illiquidity for firms that are solvency challenged. As stated earlier, a firm's solvency position is its distance-to-default (DTD), a concept

[2]Other liquidity measures, such as Amihud's illiquidity and bid-ask spread, are also tested using the same model specifications. While the overall trend in parameter estimates is similar to that using bond noise measure, the signs sometimes do not agree. This is because Amihud's illiquidity and bid-ask spread display a declining trend over our sample period (1990-2012), reflecting declining transaction costs and price impact over the years. Such declining trend causes wrong signs in the estimated parameter, because the default rate in the early period of our sample is low, relative to that during 2007-2009 financial crisis. Indeed, when using the subperiod of January 1990 - December 1997, the aggregate Amihud's measure is insignificant in our models, but becomes positive and significant (consistent with results using the bond noise measure) when using data from January 1998 - December 2012.

originated from the structural credit risk model of Merton (1974). DTD measures the number of standard deviations that the current asset value of a firm is away from its face value of debt, and it is intended to proxy the ability of a firm to meet its debt obligations. In a nutshell, DTD is the leverage adjusted by a firm's asset volatility, gauging sufficiency of a firm's equity capital to serve as the buffer in light of its volatile assets. For two equally leveraged firms, the one with a lower asset volatility will have a larger DTD, signifying its better ability to buffer a fall in its asset value.

We define $x^{(2)}_{it,13}(k)$ as:

$$x^{(2)}_{it,13}(k) = MIL_t \times 1_{\{DTD_{it} \in k \text{ deciles}\}},\ k = 1, 2, \cdots, 10 \tag{4}$$

A positive and significant coefficient of of $x^{(2)}_{it,13}(k)$ when k is low (but not when it is high) will support our hypothesis. Equation (2) with $x^{(2)}_{it,13}(k)$ will be referred to as **Model 2**.

Hypothesis 3: Market illiquidity raises PD for firms with tight funding liquidity and weak solvency

The final test is on the impact of market illiquidity for firms with different levels of funding liquidity and solvency. Specifically, we define

$$x^{(3)}_{it,13}(k_1, k_2) = MIL_t \times 1_{\{FL_{it} \in k_1 \text{ deciles } \& \ DTD_{it} \in k_2 \text{ deciles}\}},\ k_1(\text{or } k_2) = 2, 4, \cdots, 10 \tag{5}$$

We expect a positive and significant coefficient of $x^{(3)}_{it,13}(k_1, k_2)$ when FL and DTD belong to the lowest percentile group, and the impact gradually weakens as either FL or DTD improves. When $x^{(3)}_{it,13}(k_1, k_2)$ is used in equation (2), it will be referred to as **Model 3**.

For robustness, both current value and one-year moving average of FL and DTD are separately tested for each of the three hypotheses. Using current value has the advantage of reflecting the most recent information, whereas one-year moving average is less noisy and more reflective of the overall level of FL or DTD.

2.3 Data

We obtain data on US public firms from the RMI-CRI database at the National University of Singapore. This database covers over 60,000 public firms in 106 economies in Asia-Pacific, Europe, Americas, Middle East and Africa. The RMI-CRI was launched in 2009 to act as a not-for-profit counterbalancing force to the for-profit credit rating agencies (see Duan and Van Laere (2012)). The RMI-CRI database comprises three categories of data: (1) macroeconomic series, market prices and fundamental corporate data distilled from Thomson Reuters Datastream and Bloomberg Data License Back Office Product, (2) defaults and other corporate exits gathered from Bloomberg, Compustat, CRSP, Moody's reports, exchange websites and news sources, and (3) PDs and DTDs produced by the RMI-CRI using its corporate default prediction model. The third category of data are made freely accessible on the RMI-CRI website. In order to study the impact of market

Table 1: Summary statistics of covariates in the forward intensity function

This table reports summary statistics for the variables used in the forward-intensity model proposed by Duan, *et al* (2012). The sample covers 1,554,182 firm-month observations (15,121 firms, 1,314 defaults and 9,534 other corporate exits) from January 1990 to December 2012. Stock index return is the trailing one-year simple return on the S&P500 index; Interest rate is the 3-Month US Treasury yield, with the historical mean subtracted to obtain a de-meaned time series; DTD is a firm's distance-to-default; CASH/TA = (Cash plus Short-term investments) / Total Assets; NI/TA = Net income / Total Assets; SIZE= log (Firm market capitalization/Economy's median market capitalization); M/B = (Market capitalization plus total liabilities)/Total Asset; SIGMA is the idiosyncratic component of a firm's total risk associated with the stock return; Market illiquidity is the "noise" in US Treasury bonds, proposed by Hu, *et al* (2013). "Level" means the measure is computed as its one-year moving average. "Trend" means the variable is computed as its current value minus its one-year moving average.

	Mean	Std	Min	25th pctl	Median	75th pctl	Max
Stock index return	0.0891	0.1714	-0.4625	0.0204	0.0983	0.2010	0.4912
Interest rate	0.1809	2.1121	-3.3731	-1.6783	0.8571	1.7961	4.6861
DTD(Level)	3.5000	2.6164	-1.1513	1.7692	3.0171	4.6615	25.3100
DTD(Trend)	-0.0245	0.9780	-6.2844	-0.4766	0.0000	0.4498	5.1965
CASH/TA(Level)	0.1756	0.2216	0.0000	0.0253	0.0738	0.2426	0.9874
CASH/TA(Trend)	-0.0044	0.0602	-0.4357	-0.0163	-0.0009	0.0099	0.4038
NI/TA(Level)	-0.0031	0.0254	-0.3062	-0.0021	0.0016	0.0055	0.1981
NI/TA(Trend)	-0.0004	0.0203	-0.2600	-0.0017	0.0000	0.0017	0.2033
SIZE(Level)	-0.5429	1.9991	-5.7032	-2.0005	-0.6745	0.7652	6.0264
SIZE(Trend)	-0.0192	0.3301	-1.9053	-0.1551	-0.0065	0.1333	1.7890
M/B	2.0977	3.0695	0.2801	1.0240	1.3004	2.0842	54.0580
SIGMA	0.1813	0.1226	0.0295	0.0954	0.1490	0.2302	0.9814
Market illiquidity	3.3198	2.1778	0.8707	2.2310	2.8734	3.8183	18.3790

liquidity on default, we take the RMI-CRI data in the first two categories plus the DTDs in the third category.

We follow the RMI-CRI definition of default. A default event is recognized if one of the following happens: (1) bankruptcy filing, receivership, administration, liquidation or any other legal impasse to the timely settlement of interest and/or principal payments; (2) a missed or delayed payment of interest and/or principal, excluding delayed payments made within a grace period; and (3) debt restructuring or distressed exchange, in which debt holders are offered a new security or package of securities that result in a diminished financial obligation. Other corporate exits include all delistings exclusive of defaults.

For market illiquidity, we use the noise measure series available on Jun Pan's website[3], which is based on Hu, *et al* (2013) by using the CRSP Daily Treasury database to back out zero-coupon yield curves and calculates the noise measure as the root mean squared distance between the market yields and the model-implied yields. This noise measure is stated in basis points. The variable, MIL_t used in our analysis is the monthly average of daily noise measures.

While the RMI-CRI data are available for global firms up to the latest month, our study focuses on US firms in 1990-2012 due to the constraint in market illiquidity measure (only available for the US market and updated to the end of 2012). Our final sample comprises 1,554,182 firm-month observations from January 1990 to December 2012. Over this sample period, there are 15,121 public firms with 1,314 default events and 9,534 other corporate exits. The summary statistics are provided in Table 1.

3 Main Results

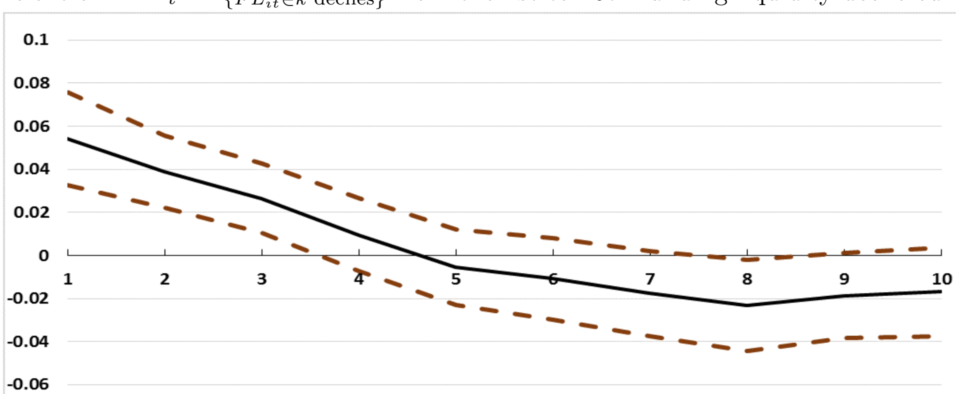
We estimate the model corresponding to each of the three hypotheses using the pseudo-maximum likelihood estimation and inference method as in Duan, *et al* (2012). Figure 1 plots the point estimate and 90% confidence interval for the coefficient of $x^{(1)}_{it,13}(k)$ as in Model 1 for the forward starting time of 11 months, where the market liquidity variable is defined relative to the percentile of individual firm's funding liquidity. Sub-figure (a) uses the current value of FL to define the indicator function $1_{\{FL_{it} \in k \text{ deciles}\}}$, whereas sub-figure (b) employs the one-year moving average of FL in the definition. With a monthly discretization, this forward default intensity function is meant for the forward period from 11 to 12 months. In both sub-figures, we see significantly positive coefficients for $x^{(1)}_{it,13}(k)$ when a firm's funding liquidity is in the bottom three deciles. Moreover, we observe a declining trend in the coefficient as funding liquidity improves, and the coefficient $x^{(1)}_{it,13}(k)$ is no longer positive and significant when funding liquidity reaches the mid-range. The results suggest that the forward default intensity is increasing in market illiquidity only for firms with little cash in hand, and support Hypothesis 1 that market liquidity plays a role in default prediction on firms with insufficient funding liquidity.

[3] `http://www.mit.edu/~junpan/`

Figure 1: Coefficient of market illiquidity as a function of a firm's funding liquidity condition

This figure displays the estimated coefficient of $MIL \times 1_{\{FL_{it} \in k \text{ deciles}\}}$ in Model 1 (corresponding to the forward starting time of 11 months) from the 1st to 10th funding liquidity decile cumulatively (i.e, $k = 1, 2, \cdots, 10$). Other covariates include: Stock index return, Interest rate, DTD(Level), DTD(Trend), CASH/TA(Level), CASH/TA(Trend), NI/TA(Level), NI/TA(Trend), SIZE(Level), SIZE(Trend), M/B and SIGMA. The definitions of these variables are in Table 1. Figure (a) uses the current value of FL in defining the indicator function $1_{\{FL_{it} \in k \text{ deciles}\}}$, whereas Figure (b) employs the one-year moving average of FL. The data sample covers 1,554,182 firm-month observations with 15,121 firms, 1,314 defaults and 9,534 other exits from January 1990 to December 2012. The solid line is for the point estimates and the dotted lines for the 90% confidence interval.

(a) Coefficient of $MIL_t \times 1_{\{FL_{it} \in k \text{ deciles}\}}$ from the 1st to 10th funding liquidity decile cumulatively



(b) Coefficient of $MIL_t \times 1_{\{FL_{it}(Level) \in k \text{ deciles}\}}$ from the 1st to 10th funding liquidity decile cumulatively

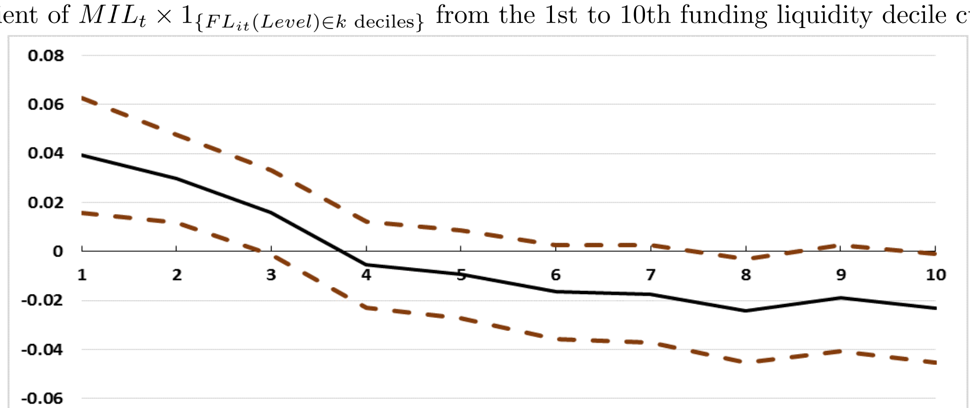
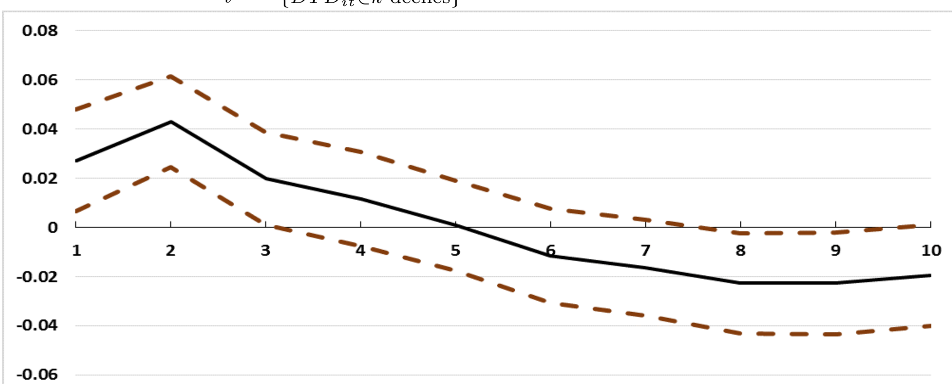


Figure 2: Coefficient of market illiquidity as a function of a firm's distance-to-default

This figure displays the estimated coefficient of $MIL \times 1_{\{DTD_{it} \in k \text{ deciles}\}}$ in Model 1 (corresponding to the forward starting time of 11 months) from the 1st to 10th funding liquidity decile cumulatively (i.e, $k = 1, 2, \cdots, 10$). Other covariates include: Stock index return, Interest rate, DTD(Level), DTD(Trend), CASH/TA(Level), CASH/TA(Trend), NI/TA(Level), NI/TA(Trend), SIZE(Level), SIZE(Trend), M/B and SIGMA. The definitions of these variables are in Table 1. Figure (a) uses the current value of DTD in defining the indicator function $1_{\{DTD_{it} \in k \text{ deciles}\}}$, whereas Figure (b) employs the one-year moving average of DTD. The data sample covers 1,554,182 firm-month observations with 15,121 firms, 1,314 defaults and 9,534 other exits from January 1990 to December 2012. The solid line is for the point estimates and the dotted lines for the 90% confidence interval.

(a) Coefficient of $MIL_t \times 1_{\{DTD_{it} \in k \text{ deciles}\}}$ from the 1st to 10th DTD decile cumulatively



(b) Coefficient of $MIL_t \times 1_{\{DTD_{it}(Level) \in k \text{ deciles}\}}$ from the 1st to 10th DTD decile cumulatively

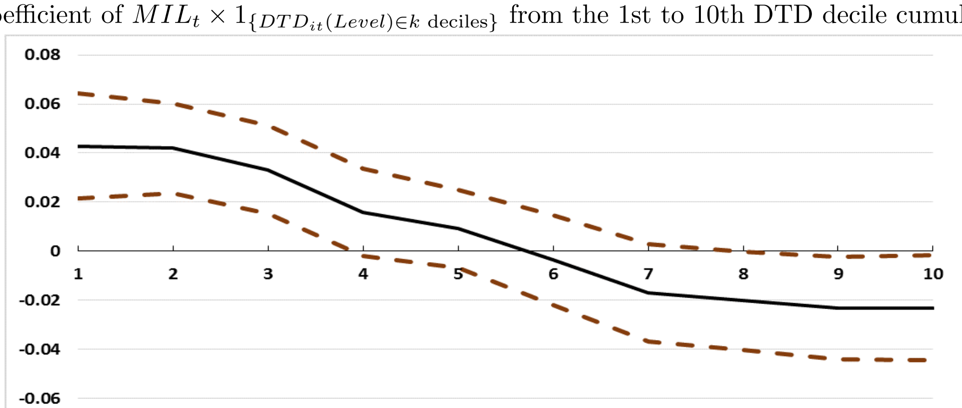


Table 2: Coefficient of market illiquidity as a function of funding liquidity and distance-to-default

This table displays the estimates for the coefficient of $MIL \times 1_{\{FL_{it} \in k_1 \text{ deciles} \ \& \ DTD_{it} \in k_2 \text{ deciles}\}}$ when this covariate is added to the forward intensity function of Duan, *et al* (2012). The intensity function corresponds to a forward period from 11 to 12 months. Other covariates in the model include stock index return, interest rate, DTD(Level), DTD(Trend), CASH/TA(Level), CASH/TA(Trend), NI/TA(Level), NI/TA(Trend), SIZE(Level), SIZE(Trend), M/B and SIGMA. The definitions of these variables are in Table 1. The sample covers 1,554,182 firm-month observations with 15,121 firms, 1,314 defaults and 9,534 other exits from January 1990 to December 2012. Panel A shows results when current values of DTD and FL are used, while Panel B is when one-year average of DTD and FL are employed. *, **, and *** indicate 10, 5, and 1 percent significance levels, respectively.

Panel A: Parameter estimates using DTD_{it} and FL_{it}					
DTD\FL	First 2 deciles	First 4 deciles	First 6 deciles	First 8 deciles	All deciles
First 2 deciles	0.0662***	0.0462***	0.0470***	0.0449***	0.0448***
First 4 deciles	0.0477***	0.0229**	0.0129	0.0099	0.0145
First 6 deciles	0.0411***	0.0140	-0.0011	-0.0118	-0.0074
First 8 deciles	0.0365***	0.0090	-0.0090	-0.0220*	-0.0177
All deciles	0.0384***	0.0096	-0.0096	-0.0203*	-0.0145
Panel B: Parameter estimates using $DTD(Level)$ and $FL(Level)$					
DTD\FL	First 2 deciles	First 4 deciles	First 6 deciles	First 8 deciles	All deciles
First 2 deciles	0.0633***	0.0374**	0.0401***	0.0448***	0.0420***
First 4 deciles	0.0448***	0.0183*	0.0159	0.0205*	0.0159
First 6 deciles	0.0380***	0.0059	-0.0001	0.0027	-0.0035
First 8 deciles	0.0297***	-0.0049	-0.0145	-0.0192	-0.0200
All deciles	0.0299***	-0.0053	-0.0165	-0.0241*	-0.0229*

Similarly, Figures 2(a)-(b) plot the estimates for the coefficient of $x^{(2)}_{it,13}(k)$ in the forward default intensity function of Model 2 where current value and one-year moving average of DTD are used respectively to define the indicator function $1_{\{DTD_{it} \in k \text{ deciles}\}}$. The results suggest that the estimate is significantly positive when a firm's solvency is in the bottom three deciles but not for those in a higher decile. A declining trend is also evident in the plot based on the one-year moving average of DTD. If current value of DTD is used in defining solvency, a small increase in the coefficient estimate occurs when one moves from the first decide to the first two deciles, but for other deciles the pattern is quite similar. This could be due to the more volatile nature of the current value of DTD for firms near default. Using one-year moving average of DTD thus provides better separation of solvency across firms that are near default.

One might notice that the estimates for the coefficient of $x^{(1)}_{it,13}(k)$ or $x^{(2)}_{it,13}(k)$ are sometimes negative when higher FL or DTD firms are included in the measurement. This may be due to an unintended adjustment role played by market illiquidity. Market illiquidity could play an adjustment role when PD is over-estimated, particularly during a crisis. This is because in an illiquid market, the lack of arbitrage capital results in "limits of arbitrage", which leads to traded prices deviating from fundamental values. Since market prices serve as an important input in calculating

PDs, a lower accuracy in market prices leads to lower accuracy of PDs. During a financial crisis, for example, SIGMA is likely over-estimated due to large noise-caused price moves. The extra noise in a crisis period in turn pushes the PDs generated by the model unnecessarily higher, and thus requires a negative adjustment to improve the model's overall fit. Moreover, Duan and Fulop (2009) showed that asset volatilities are over-estimated if trading noises are ignored. Zou (2014) further demonstrated that DTDs are under-estimated when trading noises are not factored in. By implication, PDs are likely over-estimated in a crisis period and a negative adjustment via another channel, if allowed, will certainly improve the model's fit. Market illiquidity being added to the model may just play such an adjustment role.

Table 2 shows the parameter estimates of $x_{it,13}^{(3))}(k_1, k_2)$ in the forward default intensity function by varying funding liquidity and solvency. Panel A contains results when current value of FL and DTD are used to define $1_{\{FL_{it} \in k_1 \text{ deciles } \& \ DTD_{it} \in k_2 \text{ deciles}\}}$, $k_1(\text{or } k_2) = 2, 4, \cdots, 10$, whereas Panel B is for the case that one-year moving averages of these two measures are applied in the defintion. Both cases yield similar results; that is, forward default intensity is increasing in market illiquidity only when firms' funding liquidity and/or solvency is weak. The effect becomes weaker if either the firm's funding liquidity or solvency condition improves. When both funding liquidity and solvency are solid, the parameters are no longer positively significant. Hence, market illiquidity is most helpful in default prediction when a firm's funding liquidity is low and solvency position is poor, i.e., bankruptcy risk is high.

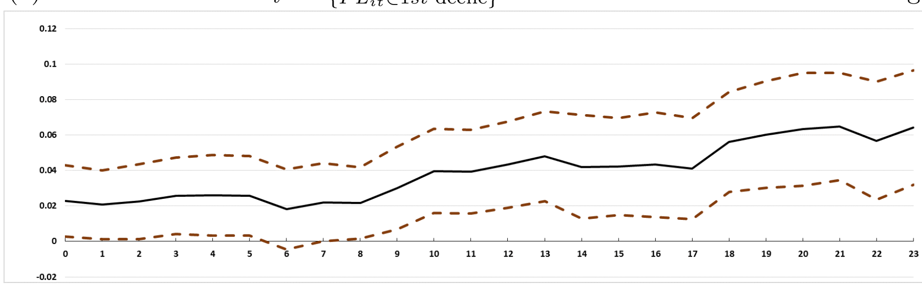
Across different prediction horizons, however, we see slightly different results. Figure 3 displays the parameter estimates for $x_{it,13}^{(1)}(k)$, $x_{it,13}^{(2)}(k)$ and $x_{it,13}^{(3)}(k_1, k_2)$ from 0 to 23 months forward starting when firms' funding liquidity and/or solvency belongs to the lowest group. The results reveal that market liquidity effect is significant from the prediction horizon of 9 months onwards for firms in the lowest decile of funding liquidity; significant from 4 to 18 months for firms in the lowest decile of solvency; and significant from 4 months onwards when both funding liquidity and solvency are the lowest. Such results indicate that market liquidity effect is not strong for very short horizons, but becomes stronger once the prediction horizon gets longer and reaches beyond a quarter. This may be due to the time needed to work out asset adjustments and/or capital raising. When market liquidity is low, working out asset adjustments and/or raising additional capital would certainly become harder.

To show economic significance of the market liquidity effect, Figure 4 plots the monthly time series of the differences in the estimated PDs (in basis points) between each of the three new models and the original forward-intensity model. For each month, we first compute the 12-month PD under one model for each firm and then the cross-sectional mean of PDs for firms with the indicator value being 1; i.e. we concentrate on firms for which market liquidity will plays a role in determining their default likelihood. We repeat this for all four models and calculate the difference between a new model and the original forward-intensity model. If the effect of market liquidity is economically trivial, then we would not expect to see large differences in the estimated PDs even for firms whose funding liquidity and/or solvency is weak. From the plots, we see large fluctuations in the PD differences over time, with the difference being larger when the market is more illiquid, or during

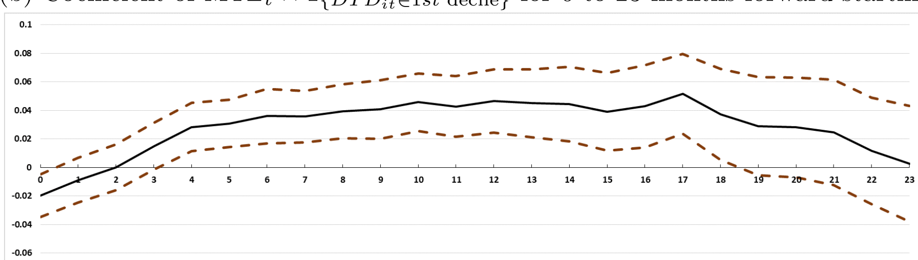
Figure 3: Coefficient of market illiquidity for different forward starting times

This figure shows the parameter estimates of $MIL \times 1_{\{FL \in 1st \text{ decile}\}}$, $MIL \times 1_{\{DTD \in 1st \text{ decile}\}}$, and $MIL_t \times 1_{\{FL_{it} \in 2 \text{ deciles } \& \ DTD_{it} \in 2 \text{ deciles}\}}$ in the forward default intensity function corresponding to different prediction horizons. Other covariates include: Stock index return, Interest rate, DTD(Level), DTD(Trend), CASH/TA(Level), CASH/TA(Trend), NI/TA(Level), NI/TA(Trend), SIZE(Level), SIZE(Trend), M/B and SIGMA. The definitions of these variables are in Table 1. One-year moving averages of FL and DTD are used in defining the indicator. The sample covers 1,554,182 firm-month observations with 15,121 firms, 1,314 default and 9,534 other exits from January 1990 to December 2012. The solid line is for the parameter estimates and the dotted lines depict the 90% confidence interval.

(a) Coefficient of $MIL_t \times 1_{\{FL_{it} \in 1st \text{ decile}\}}$ for 0 to 23 months forward starting



(b) Coefficient of $MIL_t \times 1_{\{DTD_{it} \in 1st \text{ decile}\}}$ for 0 to 23 months forward starting



(c) Coefficient of $MIL_t \times 1_{\{FL_{it} \in 2 \text{ deciles } \& \ DTD_{it} \in 2 \text{ deciles}\}}$ for 0 to 23 months forward starting

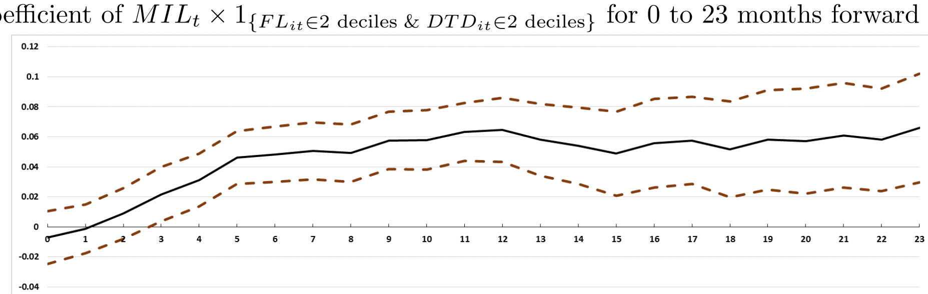
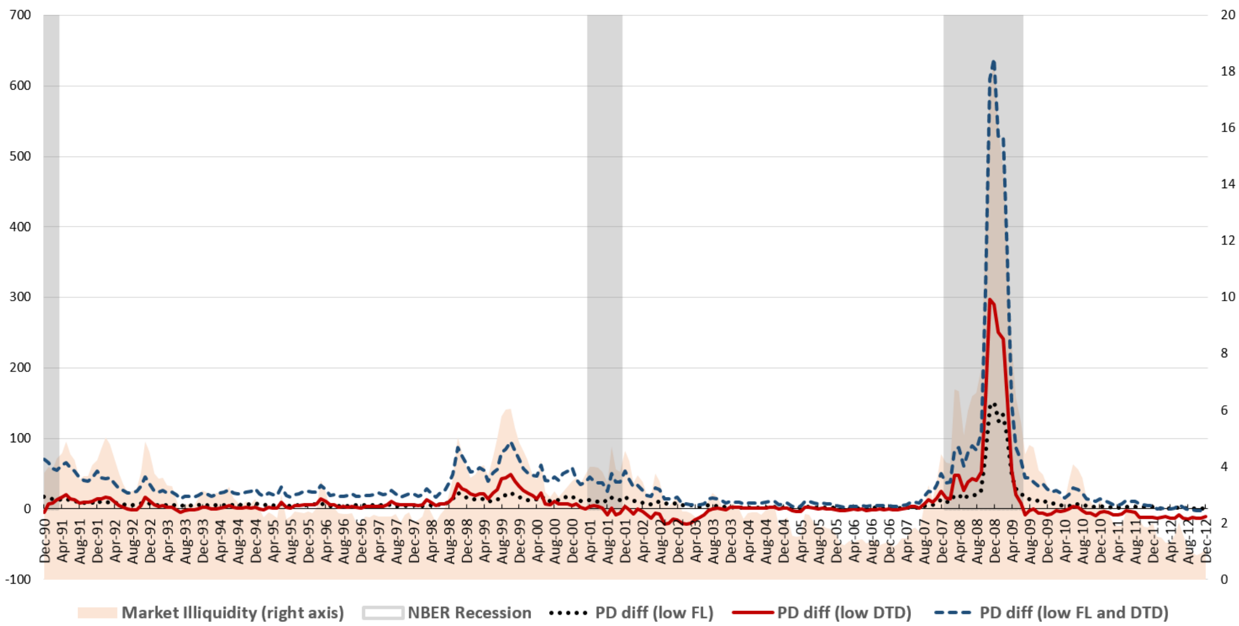


Figure 4: Monthly time series of PD differences between each of the three new models and the original forward-intensity model

This figure plots the cross-sectional average PD difference between a new model with a market illiquidity factor and the original forward-intensity model of Duan, *et al* (2012). The dotted line is the PD difference by adding $MIL \times 1_{\{FL \in 1st \text{ decile}\}}$, the solid line by adding $MIL \times 1_{\{DTD \in 1st \text{ decile}\}}$, and the dashed line by adding $MIL \times 1_{\{FL_{it} \in 2 \text{ deciles } \& \ DTD_{it} \in 2 \text{ deciles}\}}$. Grey areas mark NBER recessions. PD differences are computed only for firms in the category that the indicator function takes a value of 1. The market illiquidity measure of Hu, *et al* (2013) is plotted against the left axis. The sample covers 1,554,182 firm-month observations with 15,121 firms, 1,314 defaults and 9,534 other exits from January 1990 to December 2012. The table at the bottom is the PD implied Rating (PDiR) mapping obtained from the RMI Quarterly Credit Report, Q4/2013.

(a) Time series of PD differences (in bps)



(b) PD implied Rating (PDiR)

PDiR	Upper bound (bps)
AAA	0.16
AA	2.55
A	10.1
BBB	39.7
BB	157
B	617
CCC/C	-

a crisis (such as the LTCM episode, dotcom bubble, and more recent subprime mortgate crisis). In August 2007 when the subprime mortgage crisis first broke out, the average difference in the PDs starts to rise, and quickly soars in the aftermath of the Lehman bankruptcy and the bailout of AIG. The peak of market illiquidity coincides with the largest difference in the PDs, which is 638 bps when both funding liquidity and solvency are considered (Model 3), 297 bps when only solvency (Model 2) is considered, and 150 bps when only funding liquidity (Model 1) is used. Such differences in PDs imply that market liquidity is economically significant for default prediction, especially during a crisis.

To see how the numerical differences in PDs could have been translated into changes in credit rating, we apply the PD implied Rating (PDiR) mapping in the RMI Quarterly Credit Report, Q4/2013 to generate Figure 4(b).[4] The RMI-PDiR mapping at the bottom of Figure 4(b) is constructed in a way to match the historical default rates of different S&P credit rating categories. Under Model 3, firms with weak solvency and tight funding liquidity would have lower implied credit ratings after factoring in the market liquidity factor. Given that the mean PD difference between Model 3 and the original forward-intensity model is 38 bps, the impact for A rated firms would be a downgrade to BBB whereas for BBB rated firms, they would be downgraded to BB. Under the extreme market condition in December 2008 with the maximum change of 638 bps, the impact would be a downgrade from any investment grade to CCC/C.

If we focus on firms with tight funding liquidity (i.e., Model 1), the average increase in PDs would be 10 bps, which means that on average, a firm of AAA rating could be downgraded to A, and a firm of AA rating could be downgraded to either A or BBB. With the maximum difference of 150 bps in December 2008, the inclusion of market liquidity for firms of tight funding liquidity could result in a downgrade from AAA/AA to BBB, or from A/BBB to BB/B. Such magnitude of change in the implied credit rating is undoubtedly important to debtors and creditors alike. Under Model 2 where market illiquidity enters as a function of solvency, we see a mean PD difference of 9 bbs. An increase of 9 bps in PD implies that a firm of AAA rating would be downgraded to AA or lower, and an AA rated firm could be downgraded to A or lower. The maximum difference of 297 bps occurs in November 2008, which implies a downgrade from any rating better than B to B. Overall speaking, firms whose funding liquidity is tight or solvency is challenged will face an impact to its PD large enough to downgrade at least a notch during a non-crisis period, and much more in a crisis period.

4 Robustness of the Findings

To see whether our findings are due to the specific features of the forward-intensity model of Duan, *et al* (2012) and/or the RMI-CRI dataset employed, we now apply a less powerful but more commonly used multiperiod logistic regression model of Shumway (2001) to each of the three datasets as per Altman (1968), Zmijewski (1984) and Shumway (2001), respectively. For this robustness study, we also follow a typical practice of using annual data and excluding financial firms. Shumway (2001)

[4] Obtained from `http://rmicri.org/cms/publication/qcr/`

Table 3: Summary statistics for variables used in the logistic regression

Summary statistics for all the variables used in Altman (1968), Zmijewski (1984) and Shumway (2001) to predict corporate default. Each observation represents a particular firm in a particular year. The sample contains 84,018 firm-year observations covering 9,787 firms in 1991-2012 with 875 default events. WC/TA = working capital to total assets; RE/TA = retained earnings to total assets; EBIT/TA = earnings before interest and taxes to total assets; ME/TL = market equity to total liabilities; S/TA = sales to total assets; NI/TA = net income to total assets; TL/TA = total liabilities to total assets; CA/CL = current assets to current liabilities; Sigma is idiosyncratic standard deviation of each firm's stock returns; $r_{i,t-1} - r_{m,t-1}$ is the return of the firm in year $t-1$ minus the value-weighted CRSP NYSE/AMEX index return in year $t-1$; Relative size is the logarithm of the ratio of market capitalization to the total size of the NYSE and AMEX market; Ln(age) is the logarithm of the number of calender years that the firm's stock has been listed. All variables are winsorized at 1st and 99th percentiles.

Variable	Mean	Median	Std Dev	Minimum	Maximum	N
WC/TA	0.2809	0.2576	0.2550	-0.2456	0.8506	81680
RE/TA	-0.3695	0.0842	1.5148	-8.6424	0.8566	82994
EBIT/TA	0.0033	0.0639	0.2211	-0.9054	0.2997	83475
ME/TL	6.5427	2.1963	11.8694	0.0068	66.4325	81398
S/TA	1.1146	0.9828	0.7883	0.0059	3.7712	83912
NI/TA	-0.0464	0.0287	0.2513	-1.1076	2.7648	83911
TL/TA	0.4797	0.4758	0.2477	0.0481	1.1657	83820
CA/CL	3.1002	2.0702	3.5396	0.3498	28.5926	81674
Sigma	0.1359	0.1130	0.0901	0.0254	0.5228	76937
$r_{i,t-1} - r_{m,t-1}$	0.0764	-0.0675	0.9458	-1.3112	37.5981	76570
Relative size	-10.6624	-10.7432	2.0442	-18.0014	-2.9757	81593
Ln(age)	1.6371	1.7918	0.8744	0	3.7612	38859

showed that in light of discrete-time hazard rate models, the correct implementation of logistic regression for default prediction must involve all time series data points, including those prior to a default, so as to properly account for survivorship biases. In addition, the test statistics need to be adjusted for dependence in time series observations, i.e. a firm defaulting at t must have survived till $t-1$. In short, the number of observations used to compute test statistics should be the number of firms in the data sample, rather than the number of firm-year observations. In our robustness study, the adjusted χ^2 and p-value as suggested by Shumway (2001) are provided.

The set of all default predictors in Altman (1968), Zmijewski (1984) and Shumway (2001) are: working capital to total assets (WC/TA), retained earnings to total assets (RE/TA), earnings before interest and taxes to total assets (EBIT/TA), market equity to total liabilities (ME/TL), sales to total assets (S/TA), net income to total assets (NI/TA), total liabilities to total assets (TL/TA), current assets to current liabilities (CA/CL), idiosyncratic standard deviation of each firm's stock returns (Sigma), the return of the firm in year $t-1$ minus the value-weighted CRSP NYSE/AMEX index return in year $t-1$ ($r_{i,t-1}-r_{m,t-1}$), logarithm of the ratio of market capitalization to the total size of the NYSE and AMEX market (Relative size), and logarithm of the number of calender years the firm has been traded (Ln(age)). The accounting data are obtained from COMPUSTAT, return data from CRSP, and default events and DTD from the RMI-CRI database. Data treatment follows that of Shumway (2001). First, we lag all data by one year to ensure observability at the time of making default prediction. Second, each variable is winsorized at the 1st and 99th percentiles. Third, financial firms (SIC code from 6000 to 6799) are excluded. The sample contains 84,018 firm-year observations covering 9,787 firms in 1991-2012 with 875 default events.[5] The summary statistics are given in Table 3. All variables are of similar ranges as those in Shumway (2001) except for RE/TA and NI/TA. In our sample, these two variables have negative means, which could be due to a different sample period and the inclusion of a larger number of defaulted firms. Despite these differences, we still observe qualitatively similar parameter estimates for these default predictors.

Similar to our earlier treatment of the market liquidity variable, we add "Market illiquidity$\times$Ind" into the model, and three versions are tested. The first is to test the role of market liquidity when firms' funding liquidity is tight, for which Ind $= 1_{\{FL_{it} \in 1st \text{ decile}\}}$. Funding liquidity is calculated using the COMPUSTAT cash and short-term investment divided by total asset. The second version is when a firm's solvency is weak, i.e., Ind $= 1_{\{DTD_{it} \in 1st \text{ decile}\}}$. The third model is when both funding liquidity and solvency are weak, i.e., Ind $= 1_{\{FL_{it} \in 2 \text{ deciles } \& \ DTD_{it} \in 2 \text{ deciles}\}}$. Tables 4-6 report the logistic regression results for the three specifications along with both the unadjusted and adjusted χ^2's.

For Model 1 reported in Table 4, the parameter of market illiquidity is positive and significant for each of the three sets of default predictors. By employing Shumway's (2001) adjusted χ^2, it is only significant if the predictors are those of Shumway (2001) or Zmijewski (1984). For Model 2 reported in Table 5, however, the parameter of market illiquidity is always positive and significant for each of the three sets of default predictors, even after applying the adjusted χ^2. In the case of Model 3 reported in Table 6, the parameter of market illiquidity is also positive and significant for all specifications with or without Shuway's adjustment. The signs for other significant predictors are also consistent with the prior literature. Note that half of Altman's default predictors are statistically insignificant. But this finding is consistent with Shumway's (2001) replication study based on Altman's (1968) variables, which shows significance only for EBIT/TA and ME/TL. Significance for Zmijewski's and Shumway's variables are stronger than those reported in Shumway (2001), likely due to the bigger samples used. Taken together, we can conclude from these analyses that our earlier finding on market liquidity as a useful default predictor is robust to the use of a different econometric technique and different sets of commonly adopted default predictors.

5 Conclusions

Over a decade ago, researches like Shumway (2001) found that market information, in addition to accounting information, is informative of corporate default/bankruptcy. With the recent development in the market liquidity literature, it is natural to ask whether and how market liquidity can

[5]Firms that exited due to reasons other than default are kept in the sample and treated as survived ones for the year of exit. Such censoring control is implicit in many studies which fail to explicitly factor in other exits. Were such firms excluded from the sample, PDs would be upwardly biased.

Table 4: Logistic regression: Model 1 for Hypothesis 1 (Ind = $1_{\{FL_{it} \in 1st \text{ decile}\}}$)

This table presents the logistic regression parameter estimates for the market illiquidity variable when a firm's funding liquidity is low while controlling for Shumway's default predictors (Panel A), Zmijewski's default predictors (Panel B), and Altman's default predictors (Panel C). The variable definitions are given in Table 3. The last two columns report Shumway's (2001) adjusted χ^2 and p-value. The sample period is 1991-2012, and the sample for Panel A is comprised of 76,088 firm-year observations with 8,750 firms and 790 defaults; for Panel B 37,872 firm-year observations with 5,397 firms and 528 defaults; and for Panel C 36,307 firm-year observations with 5,083 firms and 494 defaults.

Panel A : Shumway's default predictors					
Variable	Coefficient	χ^2	p-value	Shumway adj. χ^2	Shumway adj. p-value
Intercept	-8.63	4022.35	< .0001	502.79	< .0001
Market illiquidity×Ind	0.13	24.12	< .0001	3.02	0.0825
NI/TA	-0.51	22.39	< .0001	2.80	0.0943
TL/TA	3.83	712.48	< .0001	89.06	< .0001
Relative size	-0.20	67.45	< .0001	8.43	0.0037
$r_{i,t-1} - r_{m,t-1}$	-1.89	332.95	< .0001	41.62	< .0001
Sigma	4.40	152.63	< .0001	19.08	< .0001
Panel B : Zmijewski's default predictors					
Variable	Coefficient	χ^2	p-value	Shumway adj. χ^2	Shumway adj. p-value
Intercept	-6.07	995.67	< .0001	142.24	< .0001
Market illiquidity×Ind	0.13	19.10	< .0001	2.73	0.0986
NI/TA	-1.67	240.13	< .0001	34.30	< .0001
TL/TA	3.36	329.51	< .0001	47.07	< .0001
CA/CL	-0.16	15.86	< .0001	2.27	0.1323
Ln(age)	-0.21	15.55	< .0001	2.22	0.1361
Panel C : Altman's default predictors					
Variable	Coefficient	χ^2	p-value	Shumway adj. χ^2	Shumway adj. p-value
Intercept	-3.06	517.95	< .0001	73.99	< .0001
Market illiquidity×Ind	0.07	5.15	0.0232	0.74	0.3910
WC/TA	-2.83	149.13	< .0001	21.30	< .0001
RE/TA	0.03	0.79	0.3741	0.11	0.7369
EBIT/TA	-2.99	185.09	< .0001	26.44	< .0001
ME/TL	-0.36	114.74	< .0001	16.39	< .0001
S/TA	0.02	0.11	0.7379	0.02	0.8993
Ln(age)	-0.15	6.65	0.0099	0.95	0.3298

Table 5: Logistic regression: Model 2 for Hopothesis 2 (Ind = $1_{\{DTD_{it} \in 1st \text{ decile}\}}$)

This table presents the logistic regression parameter estimates for the market illiquidity variable when a firm's distance-to-default is low while controlling for Shumway's default predictors (Panel A), Zmijewski's default predictors (Panel B), and Altman's default predictors (Panel C). The variable definitions are given in Table 3. The last two columns report Shumway's (2001) adjusted χ^2 and p-value. The sample period is 1991-2012, and the sample for Panel A is comprised of 76,088 firm-year observations with 8,750 firms and 790 defaults; for Panel B 37,872 firm-year observations with 5,397 firms and 528 defaults; and for Panel C 36,307 firm-year observations with 5,083 firms and 494 defaults.

Panel A : Shumway's default predictors

Variable	Coefficient	χ^2	p-value	Shumway adj. χ^2	Shumway adj. p-value
Intercept	-8.26	3655.03	< .0001	456.88	< .0001
Market illiquidity×Ind	0.23	147.89	< .0001	18.49	< .0001
NI/TA	-0.63	32.93	< .0001	4.12	0.0425
TL/TA	3.32	482.48	< .0001	60.31	< .0001
Relative size	-0.14	32.57	< .0001	4.07	0.0436
$r_{i,t-1} - r_{m,t-1}$	-1.72	270.33	< .0001	33.79	< .0001
Sigma	3.49	90.05	< .0001	11.26	0.0008

Panel B : Zmijewski's default predictors

Variable	Coefficient	χ^2	p-value	Shumway adj. χ^2	Shumway adj. p-value
Intercept	-6.02	990.13	< .0001	141.45	< .0001
Market illiquidity×Ind	0.35	335.11	< .0001	47.87	< .0001
NI/TA	-1.46	175.00	< .0001	25.00	< .0001
TL/TA	2.76	207.43	< .0001	29.63	< .0001
CA/CL	-0.12	10.33	0.0013	1.48	0.2243
Ln(age)	-0.26	22.93	< .0001	3.28	0.0703

Panel C : Altman's default predictors

Variable	Coefficient	χ^2	p-value	Shumway adj. χ^2	Shumway adj. p-value
Intercept	-3.60	671.56	< .0001	95.94	< .0001
Market illiquidity×Ind	0.33	264.27	< .0001	37.75	< .0001
WC/TA	-2.52	124.02	< .0001	17.72	< .0001
RE/TA	0.05	2.93	0.0869	0.42	0.5176
EBIT/TA	-2.63	136.72	< .0001	19.53	< .0001
ME/TL	-0.21	53.39	< .0001	7.63	0.0057
S/TA	-0.03	0.34	0.5592	0.05	0.8253
Ln(age)	-0.18	9.77	0.0018	1.40	0.2374

Table 6: Logistic regression: Model 3 for Hypothesis 3 (Ind = $1_{\{FL_{it} \in 2 \text{ deciles } \& \ DTD_{it} \in 2 \text{ deciles}\}}$)

This table presents the logistic regression parameter estimates for the market illiquidity variable when a firm's funding liquidity and distance-to-default are low while controlling for Shumway's default predictors (Panel A), Zmijewski's default predictors (Panel B), and Altman's default predictors (Panel C). The variable definitions are given in Table 3. The last two columns report Shumway's (2001) adjusted χ^2 and p-value. The sample period is 1991-2012, and the sample for Panel A is comprised of 76,088 firm-year observations with 8,750 firms and 790 defaults; for Panel B 37,872 firm-year observations with 5,397 firms and 528 defaults; and for Panel C 36,307 firm-year observations with 5,083 firms and 494 defaults.

Panel A : Shumway's default predictors					
Variable	Coefficient	χ^2	p-value	Shumway adj. χ^2	Shumway adj. p-value
Intercept	-8.54	3986.58	$< .0001$	498.32	$< .0001$
Market illiquidity×Ind	0.21	50.48	$< .0001$	6.31	0.0120
NI/TA	-0.53	23.49	$< .0001$	2.94	0.0866
TL/TA	3.79	694.41	$< .0001$	86.80	$< .0001$
Relative size	-0.19	58.16	$< .0001$	7.27	0.0070
$r_{i,t-1} - r_{m,t-1}$	-1.85	321.08	$< .0001$	40.13	$< .0001$
Sigma	4.29	144.71	$< .0001$	18.09	$< .0001$
Panel B : Zmijewski's default predictors					
Variable	Coefficient	χ^2	p-value	Shumway adj. χ^2	Shumway adj. p-value
Intercept	-6.03	996.07	$< .0001$	142.30	$< .0001$
Market illiquidity×Ind	0.33	91.16	$< .0001$	13.02	0.0003
NI/TA	-1.65	235.54	$< .0001$	33.65	$< .0001$
TL/TA	3.30	313.76	$< .0001$	44.82	$< .0001$
CA/CL	-0.15	15.25	$< .0001$	2.18	0.1399
Ln(age)	-0.22	16.29	$< .0001$	2.33	0.1272
Panel C : Altman's default predictors					
Variable	Coefficient	χ^2	p-value	Shumway adj. χ^2	Shumway adj. p-value
Intercept	-3.09	542.94	$< .0001$	77.56	$< .0001$
Market illiquidity×Ind	0.27	59.87	$< .0001$	8.55	0.0034
WC/TA	-2.75	141.25	$< .0001$	20.18	$< .0001$
RE/TA	0.02	0.43	0.51	0.06	0.8039
EBIT/TA	-2.93	175.03	$< .0001$	25.00	$< .0001$
ME/TL	-0.34	107.41	$< .0001$	15.34	$< .0001$
S/TA	0.01	0.01	0.92	0.00	0.9711
Ln(age)	-0.16	7.52	0.01	1.07	0.3000

help in predicting default. The paper fills this literature gap by providing fresh and interesting findings linking market liquidity and default.

Unlike commonly used variables that are related to default risk in a direct way, the role that market liquidity plays is more nuanced. First of all, we do not expect it to be useful all the time, because when a firm does not need outside capital to repay its debt, market liquidity condition should not matter to its default risk. When a firm has the need to raise external capital, market liquidity condition should matter a great deal simply because it can mitigate default risk if outside funding arrives timely and in sufficient quantity. Naturally, we form three hypotheses aiming to test the sign and significance of market liquidity for firms with different funding liquidity and solvency positions.

Utilizing the forward-intensity model of Duan, *et al* (2012) and the market illiquidity measure of Hu, *et al* (2013), our main result is that market illiquidity has a positive and significant effect in default likelihood only when a firm's funding liquidity is tight and/or its solvency position is weak. We found that aggregate market liquidity does not help as much in short-term predictions (<6 months) as in medium-term predictions (6-18 months). It is also interesting to note that the difference in default probabilities (with and without market liquidity variable) can fluctuate substantially over time, and be as large as 638 bps during the 2007-2009 financial crisis. Even in a normal period, the change in default probability can be large enough to downgrade a firm's credit rating by a notch. Finally, our conclusion is shown to be robust by employing the multiperiod logistic regression model of Shumway (2001) and by using different sets of commonly adopted default predictors.

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