Semi-parametric Pricing of Derivative Warrants

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(November 1999)

Abstract

In this paper we investigate the market pricing of derivative warrants. We couple the Black-Scholes model with a nonlinear correction function to further capture contract features. The nonlinear correction is based on the local linear kernel regression technique with time to maturity of the warrant, moneyness of the warrant and volatility of the underlying stock as the regressors. The derivative warrants written on the HSBC common stock traded in the Hong Kong Stock Exchange are used as the data sample. Our semi-parametric approach is found to substantially improve the model's ability to describe the market pricing structure of derivative warrants. The performance improvement due to the nonlinear correction is significant for both in-sample and out-of-sample settings. In addition, we find consistency in the market pricing behavior across warrants issued at different times and by different financial institutions. Specifically, we find no evidence that the identity of warrant issuer can have an effect on the pricing of warrants.

Keywords: Warrants, Black-Scholes Model, Local Linear Kernel Regression.

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1 Introduction

Since their origination in 1980's, derivative warrants have become a very popular way of repackaging securities into units more accessible to small investors. Derivative warrants are long term options issued by a company, typically an investment bank, that give the holder the right to purchase (or sell) some firm's common stock at a pre-determined price (the exercise price) on or before an expiration date[1]. Recently, derivative warrants have been issued on other underlying assets such as indices or baskets of listed equities. Derivative warrants differ from equity warrants (or sometimes referred to as corporate warrants) in an important way. Equity warrants are issued by the listed company as a part of the dividend program and/or some financing packages. When equity warrants are exercised, new common shares will be released and thus result in a dilution effect. Derivative warrants are, on the other hand, issued by a third party, and the total number of shares outstanding will not be affected due to exercise of warrants. This paper focuses on the market pricing behavior of derivative warrants.

Warrants offer investors increased gearing over holding the underlying securities in the case of call warrants, and serve as a convenient means of hedging in the case of put warrants. Indeed, derivative warrants are popular in some markets such as Germany, Switzerland and Hong Kong. Derivative warrants can be more popular than standard exchange traded options when both types of contracts are available on the same underlying asset. Such is indeed the case in Hong Kong for several Hang Seng index component stocks. Despite their apparent success, little systematic research has been devoted to the understanding of their empirical characteristics, especially in terms of their market pricing.

To price derivative warrants, it is natural to employ the option pricing theory pioneered by Black and Scholes (1973), because derivative warrants are effectively options. Like the ordinary options, the properties of a warrant are in part inherited from those of the underlying assets and in part depend on the features of the contract. The literature on equity warrants such as Schwartz (1977), Noreen and Wolfson (1981), Ferri, *et al.* (1986), and Leonard and Solt (1990) suggests that the Black-Scholes model performs as well as some more complicated pricing models for warrant pricing.[2] The conclusion reached in this body of studies seems inconsistent with the general empirical findings about the Black-Scholes model when it is applied to exchange-traded standard options. It is well known, for example, that the Black-Scholes model underprices the exchange traded options because the implied volatility is on average substantially higher than the historical or realized volatility of the underlying asset. The Black-Scholes model is also known to exhibit systematic pricing biases manifested in the phenomena such as smile/smirk and term structure of implied volatilities. The conclusion about warrant pricing may in part be a result of a genuine difference as to how warrants vis-a-vis standard options are priced in the market place, or simply due to the inadequacy of the method employed in those studies in picking up the Black-Scholes model's systematic biases. To disentangle these two causes, it is essential to employ

[1]The overwhelming majority of derivative warrants are call warrants. Since 1996, however, put warrants began to emerge.

[2]Exception to this conclusion is the study by Lauterbach and Schultz (1990). They present evidence that the Black-Scholes model is outperformed by a model that assumes a constant elasticity of variance (CEV) diffusion process for the underlying asset.

a suitable analytical tool that has the flexibility to account for the Black-Scholes model's biases if any.

In this paper, we devise a semi-parametric method for the pricing of derivative warrants. Our method couples the parametric Black-Scholes model with a non-parametric nonlinear correction function that depends on the maturity and exercise price of the warrant and the historical volatility of the underlying asset. The nonlinear correction is based on the kernel regression technique, which thus bears resemblance to the method of Aït-Sahalia (1996), Broadie, *et al.* (1996) and Aït-Sahalia and Lo (1998). Our method differs from theirs in two important ways, however. First, instead of applying kernel regression directly on option prices (or implied volatilities), we couple it with the parametric Black-Scholes model. In a way, we rely on the kernel regression to pick up the aspects of warrant pricing that the Black-Scholes model fails to capture as opposed to relying totally on the non-parametric technique to perform the pricing task. Our formulation which focuses on modeling the Black-Scholes model error is conceptually similar to that of Jacquier and Jarrow (1999), although the techniques for making the adjustment are different. Second, we have employed the local linear kernel regression method instead of the Nadaraya and Watson local constant kernel regression method used in Aït-Sahalia (1996), Broadie, *et al.* (1996) and Aït-Sahalia and Lo (1998). Our adoption of a local linear kernel regression is motivated by the fact that it has better asymptotic and boundary properties (see Fan (1992)).

Most of the previous studies on warrants concentrate on equity warrants in U.S. and Japan. In this paper, we use the data on the derivative warrants traded in Hong Kong. The issuance and trading of derivative warrants are increasing steadily in Hong Kong. In fact, Hong Kong has become the Asian leader in warrant trading and has the third largest exchange-listed warrant market in the world[3]. To date systematic empirical studies on the pricing of derivative warrants are limited. To our knowledge, there are only two studies actually deal with Hong Kong's derivative warrants. Wei (1997) relies on a lower pricing bound as a means to investigate the pricing behavior of derivative warrants traded in Hong Kong. He concludes that derivative warrants are higher than the model's prediction due to a short-sale restriction. Chang, Chang, and Lim (1998) employ an information-time option pricing model to study derivative warrants and show its performance superior to the Black-Scholes model. In this paper, we apply the semi-parametric pricing method to Hong Kong's derivative warrants. We rely on the method's flexibility in picking up data regularities without being subject to the rigidity imposed by parametric models. Our findings are expected to bring a better understanding of derivative warrant pricing in general and the Hong Kong market in particular.

Derivative warrants written on Hong Kong Shanghai Banking Corporation over the period from September, 1993 to December, 1997 are used in this study. We divide the data sample into two groups: one as the in-sample data set and the other the out-of-sample data set. Our results reveal that the Black-Scholes model when applied to derivative warrants also exhibits the same systematic pricing biases similar to those observed on the exchange-traded standard options. Based on the in-sample observations, we obtain a suitable local linear kernel regression function for correcting the Black-Scholes price. Our results indicate that the local linear kernel regression function can capture the systematic pricing patterns omitted by the Black-Scholes model. Our semi-parametric method

[3]Trading is led by Germany and Switzerland according to SEHK Regional Monitor (8/1/1997).

outperforms the Black-Scholes model even after a linear regression adjustment is added on to the Black-Scholes model. This conclusion is reached irrespective of the setting being in-sample and out-of-sample. We also find no significant difference in market pricing across derivative warrants issued by different financial institutions, even though some issues appear to have higher Black-Scholes implied volatilities. In other words, the perception of some warrant issuers have dominant influence in the market place and their issues command a higher premium is more likely due to the difference in contract specifications.

The remainder of this paper is organized as follows. An exposition of the semi-parametric pricing method is given in Section 2. The derivative warrants data set is described in Section 3. Empirical results are summarized in three categories and presented in Section 4. Finally, Section 5 concludes the paper.

2 Semi-parametric pricing method

Conceptually, we view a warrant price consisting of two components – an intrinsic value and a measurement error. Specifically, the warrant price c_t at time t is written as

$$c_t = P(S_t, X, T-t, r_t, Z_t) + \epsilon_t \tag{1}$$

where $P(S_t, X, T-t, r_t, Z_t)$ stands for the intrinsic valuation function of the warrant and ϵ_t denotes the measurement error. Naturally, the intrinsic value must be a function of S_t (the underlying stock price), X (the exercise price of the warrant), $T-t$ (the maturity of the warrant with T being the expiration date), r_t (the risk-free interest rate), and Z_t (a vector of state variables that could potentially affect the price of warrant). State variables may include the underlying stock prices prior to t, as, for instance, in a non-Markovian pricing setting or the local volatility that appears in the GARCH and stochastic volatility models. State variables may also include past dividends and interest rates.

The exact intrinsic valuation function is unknown in practice. Any implementation, either parametric or non-parametric, thus calls for a model in its place. Although parametric models are generally easier to implement and can be used for extrapolation, they are more prone to systematic modeling errors. Non-parametric models, though flexible, are more data intensive and cannot be reliably used for situations that the available data have not yet covered. In other words, its applicability for extrapolation is a suspect. Moreover, implementing non-parametric techniques typically faces the difficulty of identifying a suitable and small set of factors that are critical to the performance of a model. Using a large set of factors has to be ruled out due to the "curse of dimensionality." In light of these considerations, we decompose the intrinsic value into two parts with the first being a parametric model, specifically the Black-Scholes model, and the second being the model error captured by a non-parametric technique. Specifically, the warrant price is

$$c_t = B(S_t, X, T-t, r_t) + \eta_t + \epsilon_t \tag{2}$$

where $B(S_t, X, T-t, r_t)$ is the Black-Scholes price for the warrant and η_t is the model error. The above conceptual set-up bears resemblance to that of Jacquier and Jarrow (1999), although the modeling techniques involved are quite different.[4] In this paper, we use the kernel regression technique to model η_t by relating it to contract features and historical volatility of the underlying asset.

Since measurement errors are likely to depend on the magnitude of warrant prices, we thus employ the kernel regression technique to model the relative model error. In other words,

$$\eta_t^* = \frac{\eta_t}{c_t} = g(S_t, X, T-t, r_t, Z_t), \tag{3}$$

or

$$\frac{c_t - B(S_t, X, T-t, r_t)}{c_t} = g(S_t, X, T-t, r_t, Z_t) + \epsilon_t^*. \tag{4}$$

where ϵ_t^* is the measurement error normalized by the warrant price. The most well-known kernel regression method is the Nadaraya-Watson estimator (Nadaraya, 1964; Watson, 1964). We have, however, chosen to use a local linear kernel regression (LLKR) to model $g(\cdot)$ because it has been shown to have better asymptotic and boundary properties (see Fan (1992)). We now provide a brief description about this method and its relation to the Nadaraya-Watson estimator.

Let the observed data be a sequence of $(d+1)$-dimensional vectors, denoted by $\{x_j, y_j;\ j = 1, ..., n\}$ where x_j is a d-dimensional vector. Then, LLKR relying on solving the following quadratic programming problem:

$$\min_{\alpha_h(x),\beta_h(x)} \sum_{j=1}^{n} \{y_j - \alpha_h(x) - \beta_h(x)\cdot(x_j - x)\}^2 K(x_j - x; h) \tag{5}$$

where $K: \Re^d \rightarrow \Re$ is a kernel function and $h \in (\Re^+)^d$ is the bandwidth vector. The LLKR estimator for y at the point x is the optimizer $\hat{\alpha}_h(x)$, as it defines the position of the local regression line at the point x. The kernel function $K(z; h)$ is generally a smooth positive function which peaks at 0 and decreases monotonically as z increases in size. It effectively acts as a weighting scheme to ensure that more weights are given to the observations whose covariate value x_j lie closer to the point x. The minimization problem must be solved repeatedly for different x. In other words, LLKR is a procedure of solving weighted linear regression repeatedly with different regression coefficients for different points of interest. The Nadaraya-Watson method cast in terms of the above minimization problem is to set $\beta_h(x) = 0$. In essence, it is a local constant kernel regression. The LLKR estimator, when x is one-dimensional, has the following explicit solution:

$$\hat{\alpha}_h(x) = \frac{1}{n}\sum_{j=1}^{n} \frac{\{s_2(x;h) - s_1(x;h)(x_j - x)\}K(x_j - x; h)y_j}{s_2(x;h)s_0(x;h) - s_1(x;h)^2}$$

where $s_r(x;h) = \frac{1}{n}\sum_{j=1}^{n}(x_j - x)^r K(x_j - x; h)$. The explicit solution can also be obtained for higher dimensional cases using matrix notation.

One of the crucial points in applying kernel regression is the choice of the bandwidth h, which controls the relative importance of neighboring and distant points. The larger the bandwidth is, the

[4] Jacquier and Jarrow (1999) employ a Bayesian technique which requires the model error to be linear function of some variables and assumes a specific density function for the measurement error.

smaller are the weights applying to neighboring points, which in turn causes the resulting estimator to miss the details in the curvature of the data. As the bandwidth decreases, the estimator begins to track the data more closely and eventually ends up interpolating the observed points. A proper trade-off is the essence of bandwidth selection. There are situations where one may prefer to choose the bandwidth subjectively by examining how the resulting estimator behaves. This would involve looking at several estimators over a range of bandwidths and selecting the one that seems to be the best. A more objective approach is to choose the bandwidth automatically using some sensible objective criterion. In this paper, we employ a variant of the cross-validation method. Cross-validation is to select the bandwidth based on minimizing the mean squared error between the realized value and the estimated one. This criterion has an advantage over other distance measures because it is quite tractable analytically.

To measure the accuracy of the estimator $\hat{\alpha}_h$, based on a particular bandwidth h, we calculate the theoretical mean squared error $d(\hat{\alpha}_h, \alpha)$ by integrating over all potential realizations of x as follows:

$$d(\hat{\alpha}_h, \alpha) = \int [\alpha(x) - \hat{\alpha}_h(x)]^2 w(x) p(x) dx \tag{6}$$

where $p(x)$ is the density function governing the occurrence of x, $w(x)$ is the weighting function used in the kernel regression. The optimal bandwidth is in principle the one that minimizes the mean squared error. In practice, it cannot be done without finding substitutes for $\alpha(x)$ and $p(x)$. Typically, the implementation uses a "leave-one-out" approach. In other words, it is to find an h that minimizes its empirically equivalent quantity:

$$n^{-1} \sum_{i=1}^{n} [y_i - \alpha_h^*(x_i)]^2 w(x_i) \tag{7}$$

where $\alpha_h^*(x_i)$ is the "leave-one-out" kernel estimator, i.e., the estimator for the observation at x_i by specifically excluding (x_i, y_i) from the data sample. In this paper, we employ a variant of the "leave-one-out" method because it is too computationally intensive. Our cross-validation method involves setting aside a fixed set of data points for the cross-validation purpose. We use the remaining data points to come up with the kernel estimate for every point in this fixed cross-validation data set. The mean squared error is then calculated over all points in this fixed cross-validation data set. In a sense, our cross-validation method is a "leave-one-fixed-set-out" procedure. The specific fixed set used in this paper will be described later.

In selecting the regressors to be used in the kernel regression, we also face the so-called "curse of dimensionality" problem. Loosely speaking, the higher the dimension of x is, the sparser is a fixed set of data points scattered in the relevant domain. There are many potential regressors in our specific application. To decrease the dimension of the regressors, we combine some variables; for example, using the exercise price to stock price ratio X/S_t is a natural way of reducing the number of regressors. We have discarded interest rate variable from $g(\cdot)$ because our experiments reveal that it does not enhance the performance of the kernel regression in any meaningful way. With regard to other state variables, we have found that the historical volatility plays an important role in explaining the derivative warrant pricing. The work by Broadie, *et al.* (1996) using Nadaraya-Watson kernel regression also suggests the importance of including volatility in the regression function. The historical volatility can in a way be viewed as a proxy, in the absence of a definitive stochastic volatility model, to intuitively reflect the random volatility phenomenon exhibited prominently in equity markets. To summarize, our final LLKR function contains three regressors: the normalized exercise price and maturity of the warrant and the historical volatility of the underlying stock.

3 Data

Our data set consists of all 59 derivative warrants series written on the common shares of Hong Kong and Shanghai Banking Corporation Limited (hereafter, HSBC[5]) listed on the Hong Kong Stock Exchange between September, 1993 and December, 1997. During this period, HSBC derivative warrants with different maturities and exercise prices had been issued by 19 different financial institutions. We select HSBC derivative warrants because HSBC was and continues to be the underlying stock preferred by financial institutions in issuing derivative warrants. This preference mainly reflects the dominant role of HSBC in the Hang Seng index, a key Hong Kong equity market index. HSBC common stocks constitute about 10% of the Hang Seng index in terms of market capitalization. With the high liquidity of the underlying stock, the HSBC derivative warrants are actively traded in the market. Table-1 provides a summary of the characteristics of these warrants. It is evident from this table that many financial institutions repeatedly issued derivative warrants on the HSBC stock during the sample period.

Insert Table-1

The information about warrant exercise prices, expiration date, conversion ratios, dividends is compiled from various issues of "South China Morning Post" and "Hong Kong Economic Journal Monthly". The units issued at initial listing, issue price and issue date are obtained from Stock Exchange Fact Books (Hong Kong Stock Exchange, 1992 to 1997). The daily warrant closing prices and HSBC closing prices are retrieved from the Trade Record (Equity) of the Hong Kong Stock Exchange. Figure-1 presents the prices and historical volatilities (based on the preceding year's daily returns) of the HSBC stock during the sample period. It is clear from this plot that our sample period covers a wide range of stock prices and volatilities. The conclusions reached later in the paper are therefore likely to be applicable to a wide range of market conditions. Since there is no guarantee that the closing stock price is recorded at the same time as the closing transaction for each warrant, we use the best bid-ask average of the stock price traded at a warrant's closing transaction time as the corresponding underlying stock price for that particular warrant[6]. We proxy the risk-free interest rate by using the Hong Kong Interbank Offering Rate (HIBOR) available on Datastream. The risk-free rate for any particular maturity, if it is unavailable, is obtained by interpolating the two HIBORs whose maturities straddle the target.

[5]HSBC was founded in 1865 in Shanghai. After over one hundred years of development, the bank has become one of the biggest banks in the world. HSBC is currently a banking network with presence in 75 countries. Its shares are listed on both London and Hong Kong Stock Exchanges.

[6]Hong Kong Stock Exchange uses electronic limit-order book to trade stocks and derivative warrants. Every traded security, either a common share or derivative warrant, is assigned a numerical code for trading purposes.

Insert Figure-1

Two exclusionary criteria are applied to the data set. First, we eliminate warrants with less than one week to expiration. The shorter-term warrants have relatively small time premiums, hence the estimation of model price is extremely sensitive to possible measurement errors. In addition, these shorter-term warrants become inactively traded as a part of winding down process by the issuing financial institutions. Second, observations which violate a basic arbitrage pricing bound are excluded. We use the simple lower bound for a call option, i.e., $S_t - Ke^{-rT}$ where S_t is the dividend-adjusted stock price, to exclude any observation with a warrant price lower than this bound. These exclusions leave us with 9128 observations.

4 Empirical Results

Our empirical findings are organized into three groups. First, we study the performance of the Black-Scholes model. Second, we add a linear regression adjustment factor to the Black-Scholes model to correct for biases if any. Finally, the empirical results based on the semi-parametric method are presented.

4.1 Performance of the Black-Scholes model

In order to apply the Black-Scholes model, we need a volatility estimate. In some applications, it is quite common to see that the implied volatility derived from some options is used to price other options. The implied volatility is, however, unsuitable for our purpose of pricing derivative warrants because we want to understand how the market warrant price is determined relative to the underlying stock. We thus use the historical volatility calculated according to the Black-Scholes assumption of geometric Brownian motion. Specifically, an n-day historical volatility at time t, i.e., σ_t, is calculated as follows:

$$\sigma_t^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(\ln\frac{S_{t+1-i}}{S_{t-i}} - \frac{1}{n}\sum_{j=1}^{n}\ln\frac{S_{t+1-j}}{S_{t-j}}\right)^2. \tag{8}$$

In a strict sense of the Black-Scholes model, the larger n is, the better is the quality of σ_t. The reality is different, however. Since volatilities are known to change over time, a fact contrary to the prescription of the Black-Scholes model, too large an n is likely to cause a loss of critical information about the market condition embedded in the historical volatility. We thus use four different time lengths in calculating the historical volatility – 1 month, 3 months, 6 months and 1 year.

Since all HSBC derivative warrants in our sample are of American type, we use the binomial tree method to compute the warrant's Black-Scholes price. We construct a 1000-step binomial tree and adjust for cash dividends during the life of every warrant.

Figure-2 plots out the time series of the percentage pricing errors of the Black-Scholes model. The Black-Scholes prices used to generated this plot are computed with the historical volatility using the preceding year's daily returns. It is evident that the Black-Scholes model tends to underprice these warrants. This result suggests that the Black-Scholes implied volatility inferred from the warrant price will in general be higher than the historical volatility, which is consistent with the well-known phenomenon for the exchange-traded standard options. Combining Figures 1 and 2, we see that during the high volatility periods, the Black-Scholes model actually overprices warrants. This result is hardly surprising because a higher historical volatility, *ceteris paribus*, tends to cause the model price to be higher. Such an observation also points toward the importance of including historical volatility as a regressor in constructing the correction term.

Insert Figure-2

In Table-2, the average percentage pricing differences between the Black-Scholes model price and the market price are reported for each individual warrant series. These results reconfirm the earlier observation based on the plot; that is, the Black-Scholes model generally underprices derivative warrants. Table-2 reveals that out of the 59 warrant series, 58 are underpriced by the Black-Scholes model, and the range is from 2% to 48%. The t-statistics indicate that 34 of them are significant at the 5% level and 24 are highly significant at the 1% level.

Insert Table-2

The previous evidence exists on the mispricing of Hong Kong's warrants by the Black-Scholes model, but the interpretation in these papers tends to question the efficiency of the market rather than the reliability of the Black-Scholes model. Chan and Kwok (1991), for example, study Hong Kong's equity warrants and find a similar mispricing pattern. They contend that the Hong Kong equity warrant prices are not rationally determined because warrant markets are thin. Wei (1997) concludes after analyzing Hong Kong's derivative warrants that warrants' market prices are biased upward, and he attributes this result to the short-sale restriction on derivative warrants.

Instead of questioning market efficiency, this paper explores an alternative avenue of pinning the problem on the Black-Scholes model. We first need to understand better the nature of the mispricing by the Black-Scholes model before a non-parametric corrective measure can be formulated. The past study on exchange-traded standard options suggest that systematic biases mainly surface in two dimensions - maturity and exercise price. In Table-3, the mean percentage errors (MPE), mean squared percentage errors (MSPE) and mean absolute percentage errors (MAPE) exhibited by the Black-Scholes model are categorized according to these two main attributes. This table contains three panels. Panel A contains the results for the whole data set. To study the effects caused by the recent Asian financial crisis, we divide the sample into two subperiods with the data before August 14, 1997 reported in Panel B and the remaining ones in Panel C. As was mentioned earlier, we use four different time lengths to calculate the historical volatility of the underlying stock. The results are thus reported for each one of these four measures.

Insert Table-3

Table-3 indicates that MPEs are generally positive and MSPEs after the financial crisis are particularly high. We see from this table that there is a strong positive relation between MSPE and moneyness measured by X/S_t. This relation exists for all maturity categories and for both before and after the Asian financial crisis. It is also true for four different measures of historical volatility. MSPEs generally increase with time to maturity for in-the-money warrants. But for at-the-money and out-of-the-money warrants, MSPEs for shorter maturities are generally higher than those for longer maturities. We note that using the MAPE criterion leads to similar conclusions, which implies that our conclusions are not due to a few outliers.

4.2 The Black-Scholes model with a linear regression correction

Since the Black-Scholes model exhibits clear systematic pricing biases, it may be possible to correct these biases using a linear regression. We thus consider four linear correction models:

Model 1: $\eta_t^* = a_0 + a_1\tau + a_2(X/S_t) + a_3D_{fc}$

Model 2: $\eta_t^* = a_0 + a_1\tau + a_2(X/S_t) + a_3D_{fc} + a_4\tau^2 + a_5(X/S_t)^2 + a_6\tau(X/S_t)$

Model 3: $\eta_t^* = a_0 + a_1\tau + a_2(X/S_t) + a_3D_{fc} + a_4\tau^2 + a_5(X/S_t)^2 + a_6\tau(X/S_t) + a_7\sigma_t$

Model 4: $\eta_t^* = a_0 + a_1\tau + a_2(X/S_t) + a_3D_{fc} + a_4\tau^2 + a_5(X/S_t)^2 + a_6\tau(X/S_t) + a_7\sigma_t + a_8\sigma_t^2$.

Model 1 attempts to capture the variation in the percentage mispricing (η_t^*) attributable to time-to-maturity (τ), degree of moneyness (X/S_t) and the financial crisis (D_{fc} is a dummy variable with a value of 1 if the observation is after the financial crisis and 0 if the observation is before the financial crisis). In Model 2, we add quadratic terms to the regression, hoping to capture nonlinearity in the Black-Scholes mispricing. We add the volatility term (σ_t) to the regression in Model 3 because our earlier results suggest a strong relationship between pricing errors and volatilities. Model 4 allows for volatility to enter in a quadratic form.

The regression results using all observations ($N = 9128$) are summarized in Table-4. We learn from these results that the percentage pricing error can be in part explained away by employing a linear regression correction. It appears that a model with quadratic terms performs better than the one without. For example, Model 4 delivers the best performance based on the R^2 adjusted for degrees of freedom regardless of which historical volatility measure is used. Among all, the best model appears to be Model 4 using one month of daily returns in computing the historical volatility. In this case, more than 60% of the percentage pricing errors of the Black-Scholes model has been eliminated.

Insert Table-4

The t-statistics show a strong relation between the percentage mispricing of the Black-Scholes model and the individual regressors. Interestingly, the dummy variable for the Asian financial crisis is also significant, perhaps suggesting that the market structure has changed after the financial crisis. The signs of these regression coefficients are not stable, however. This is true across models or using different historical volatility measures.

A simple correction model such as Model 1 (using 1 month or 3 months of daily data for the historical volatility) implies that the warrant with a longer time to maturity and deeper in the money tends to be underpriced more by the Black-Scholes model. Upon adding quadratic terms, i.e., Models 2, 3 and 4, the explanatory power of the model increases, but its interpretation becomes less clear. The fact that the quadratic terms are significant suggests that the relationship between the percentage pricing errors and these explanatory variables are nonlinear. In the case of Model 3 where historical volatility is added to the regression function, a significantly negative relationship appears. In fact, its magnitude decreases with the number of daily returns used in the historical volatility calculation. This result is actually not at all surprising. When historical volatility is high, it increases the Black-Scholes model value for the warrant. The Black-Scholes pricing error thus needs a downward correction in comparison to a lower historical volatility. If we let the number of daily returns used in calculating historical volatility increase to some point, then the information content in the historical volatility will begin to deteriorate and gradually loses its explanatory power.

The overall performance of the linear regression correction is best seen in the time series plot presented in Figure-2 (the middle plot) where Model 4 and 3-month historical volatilities are used. It is clear from this plot that the benefit of a linear regression correction almost entirely hinges on its correction of the overall level of underpricing by the Black-Scholes model. In other words, the pricing errors after the linear correction become centered around zero. Comparing the plots with and without the linear correction, it appears that other aspects of the bias are little affected.

Although the linear regression correction can gain explanatory power, it may simply be a result of in-sample over-fitting. From an application perspective, the real test rests with its performance in an out-of-sample setting. We thus divide the data sample into two groups with one treated as the in-sample data set and the other out-of-sample data set. The in-sample data set is actually further divided into two subgroups. This further division is unnecessary for this part of analysis, but it will be needed for the bandwidth selection in the case of the local linear kernel regression to be conducted later. For consistency and avoiding confusion, we thus maintain the same grouping throughout. The out-of-sample analysis can be conducted in two different ways. First, we construct a model using some warrant series and use the same system to price other warrants (i.e., across warrants), or we can construct the model using warrants in an earlier time period and use it to price warrants in the later period (i.e., across time periods). We will conduct both types of out-of-sample analysis.

For the across warrants analysis, we divide up 59 HSBC derivative warrants into three groups – 30 warrant series in S_1, 14 in S_2, and 15 in S_3. All three groups are well balanced and cover the whole sample periods. For the across time periods analysis, the sample is divided into three subperiods – September, 1993 to December, 1995 in T_1, January, 1996 to March, 1997 in T_2, after March 1997 in T_3. S_1 and S_2 together are viewed as the in-sample data set and S_3 as the out-of-sample data set for the across warrants analysis. Similarly, T_1 and T_2 together are viewed as the in-sample data set and T_3 as the out-of-sample data set for the across time periods analysis.

The results in Table-5 are based on Model 4 as the linear regression correction. In this table, the ratio of MSPEs for the Black-Scholes model with and without the linear regression correction is the focus. If the ratio is less than one, it suggests that the correction reduces the percentage pricing error of the Black-Scholes model. The overall error reduction is much larger when the linear regression is applied across warrants rather than across time periods. The linear correction is, of course, expected to improve the overall in-sample performance, which is indeed the case for $S_1 + S_2$ and $T_1 + T_2$. For a subcategory of the in-sample data set, the linear correction need not have a better performance, however. Interestingly, such situations do occur, for example, for in-the-money subcategory of $S_1 + S_2$. This result suggests that the linear regression does not perform the correction uniformly. In fact, the correction may make the model price worse for some categories of warrants even it is used in-sample.

Insert Table-5

When the performance is cast in the out-of-sample context, the linear regression correction in the case of the across time periods analysis becomes significantly poorer. For the across warrants analysis, however, both in-sample and out-of-sample performances are about the same. Such a result suggests that mispricings in different warrant series over the same time period are roughly the same. The linear regression can be used to correct the Black-Scholes biases if the biases of some warrant series over the same time period are known. In contrast to the across time periods analysis, this type of out-of-sample finding is less useful because future warrant prices on a different warrant series are not actually available at the time of making a correction.

It is reasonably to think that the Asian financial crisis may have distorted our results. We thus restrict our attention to the sample period before August 14, 1997 and repeat the analysis. The results in Table-5 suggest that the performance of the linear regression correction in the across warrants analysis deteriorates significantly both in-sample and out-of-sample. For the across time periods analysis, the out-of-sample performance actually improves. Indeed, the period of Asian financial crisis has an important influence on our results.

4.3 The semi-parametric pricing method

Our semi-parametric pricing method couples the Black-Scholes model with a LLKR function to correct the Black-Scholes pricing biases. As discussed earlier in Section 2, LLKR needs a bandwidth for every regressor. The bandwidths for the three regressors of our model are to be selected by a "leave-one-fixed-set-out" cross-validation method. The fixed set is S_2 for the across warrants analysis and T_2 for the across time periods analysis. This explains why we earlier divided the in-sample data set into two groups. Using the cross-validation criterion, we obtain the optimal bandwidths for three regressors under various settings. The results are reported in Table-6. The optimal bandwidths obtained this way are then used in both the in-sample and out-of-sample analyses.

Insert Table-6

The overall performance of the semi-parametric pricing method is best seen in the bottom plot of Figure-2 in which the pricing errors are much smaller than those under the Black-Scholes model with or without the linear regression correction (middle plot and top plots in Figure-2). Similar to the analysis in the preceding subsection, we also conduct the in-sample and out-of-sample analyses and examine the ratio of the percentage pricing errors under the semi-parametric model and the Black-Scholes model. The results are reported in Table-7. It is clear by comparing this table with Table-6, the LLKR correction works much better than the linear regression correction. The in-sample result is not at all surprising because the LLKR is a more flexible way of capturing data regularities. The out-of-sample superior performance suggests that it is not simply caused by an in-sample over-fitting. The most significant difference between the two correction methods is the result pertaining to the across time periods analysis. The LLKR correction works very well both in-sample and out-of-sample, whereas the linear regression performs rather poorly. As argued earlier, this is a more meaningful out-of-sample setting. In other words, the LLKR correction dominates the linear regression method and particularly so for the most relevant situation.

Insert Table-7

The Asian financial crisis was found to distort the earlier result for the linear regression correction. In contrast, we find no similar distortion with the LLKR method. Restricting the attention to the data set before the Asian financial crisis does not materially change our conclusion about the semi-parametric pricing method. It suggests that the nature of the nonlinearity is too complex to be captured by a simple inclusion of some quadratic terms in the regression function. The non-parametric method provides a simple and reliable way of capturing the complex functional relationship.

The analyses so far are restricted to relative performance based on ratios of the aggregate percentage pricing errors under different methods. It will be informative to know the distributions of the percentage pricing errors under different models. The out-of-sample results are summarized in Figure-3 (across warrants analysis) and Figure-4 (across time periods analysis). These plots provide various percentiles for the pricing errors. It is clear from these plots that the Black-Scholes model is downward biased. The linear correction can mitigate to some extent the pricing error, but its effectiveness is no way close to that of the LLKR. This is true for different categories of warrants and for both the across warrants and across time periods applications.

Insert Figure-3 and Figure-4

Derivative warrants on the same underlying stock may be issued by different issuers at the same time or at different times. The precise terms and conditions of derivative warrants often differ across issuers to reflect the market condition at the time of issuance and/or the issuer's general preferences. To protect the interests of the warrant holders, the regulations require warrant issuers to have adequate financial reserve or to secure a guarantee from the parent company. The active warrant issuers in Hong Kong are mostly European and American financial houses[7]. Some warrant issuers are dominant players in the warrant market, and they may, as a result, command a market power that can secure a premium over their competitors; for example, investors may infer from the reputation of these warrant issuers that their warrant series are of a higher liquidity. This potential reputation effect is, however, intertwined with the difference in the contractual terms of warrants. Since our semi-parametric pricing method can remove the effect of the contractual terms quite well, it thus provides an "uncontaminated" way to examine the reputation effect. We perform a multiple regression analysis with the dependent variable being either the percentage pricing error of the Black-Scholes model or that of the semi-parametric pricing method. The explanatory variables

[7]Peregrine, a Hong Kong based financial house, was very active in derivative warrants until it went bankrupt at the height of the Asian financial crisis.

are 17 dummy variables to reflect there were 18 warrant issuers in the period before the Asian financial crisis. Our results suggest that the identity of the issuer does predict the Black-Scholes pricing error; for example, the warrants issued by BZW, Peregrine and Robert Fleming tend to have a higher market price. Using the pricing error from the semi-parametric model, however, the conclusion is different. The result actually suggests that no issuer can command a higher price. This is true using the bandwidth which is selected based on either the across warrants or across time periods sample division method. We can thus interpret our earlier significance result simply as a failure of the Black-Scholes model to properly account for differences in contract specifications. In short, it should be regarded as a reflection of the inadequacy of the Black-Scholes model in pricing derivative warrants rather than a genuine issuer effect.

Insert Table-8

5 Conclusion

In this paper we devise a semi-parametric pricing method by coupling the Black-Scholes model with a local linear kernel regression function. We use this method to conduct an empirical study of the derivative warrants on the HSBC common stock. We find that the semi-parametric method works well both in-sample and out-of-sample. We also find no evidence that the identity of warrant issuer can affect the market pricing of derivative warrants.

There is a widely held belief that derivative warrants are overpriced. The fact that so many financial houses join in to issue them seems to suggest handsome profits in these activities. The short-sale restriction imposed on these warrants also prevents selling pressure to materialize quickly, and hence a possibility of overpricing exists. A seemingly more direct evidence though is the result that the market prices of derivative warrants are significantly higher than the values predicted by the Black-Scholes model. In this study, we have refuted the evidence of overpricing based on the Black-Scholes model. But we cannot say that derivative warrants are not overpriced. We can conclude, however, that if overpricing occurs, it is fairly stable over time and across different warrant series. A future analysis linking derivative warrants to exchange-traded standard options on the same underlying stock should have the potential to answer this question.

6 References

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Table-1: Descriptive statistics for the HSBC derivative warrants.

Code	Warrant Name	Listing Date	Expire Date	Conversion Ratio	Exercise Price (HKD)	Units Issued (million)	Issue Price (HKD)	Issuer
477	HSBC 94 (BZW)	08/12/92	06/17/94	0.1	45.5	400	1.1	Barclays de Zoete Wedd Warrants Ltd.
589	HSBC 94 (Harvest)	10/15/92	04/15/94	0.1	56.5	40	1.09	Harvest Top Investment Ltd.
653	HSBC 94 (Ford Deluxe)	09/24/92	09/24/94	0.1	54	80	1.35	Ford Deluxe Investment Ltd.
1115	HSBC 95 (ML)	07/06/94	12/08/95	0.1	87	100	1.914	Merrill Lynch International & Co. C.V.
573	HSBC 95 (Peregn)	03/24/93	02/20/95	0.1	63.5	250	1.35	Peregrine Derivatives Ltd.
58	HSBC 95 (SBC)	09/09/93	08/10/95	0.1	81.5	120	1.71	Swiss Bank Corp., HK
1116	HSBC 95 (SBC)	07/07/94	12/08/95	0.1	69.5	100	2.9	Swiss Bank Corp., HK
768	HSBC 95 (Peregn)	11/10/93	03/20/95	0.1	79.5	100	1.57	Peregrine Derivatives Ltd.
424	HSBC 96 (BZW)	03/10/94	01/31/96	0.1	120	250	2.98	Barclays de Zoete Wedd Warrants Ltd.
630	HSBC 96 (Peregn)	02/23/94	01/17/96	0.1	109	140	3.08	Peregrine Derivatives Ltd.
682	HSBC 96 (R Flem)	03/03/94	02/02/96	0.1	131	300	3.075	Robert Fleming & Co. Ltd.
515	HSBC 96 (SBC)	06/15/95	06/21/96	0.1	88.25	50	2.13	Swiss Bank Corp., HK
1226	HSBC 97 (MS)	03/13/95	02/10/97	0.1	72.45	50	2.45	Morgan Stanley (Jersey) Ltd.
932	HSBC 97 (BZW)	08/16/95	01/22/97	0.1	91.8	50	2.39	Barclays de Zoete Wedd Warrants Ltd.
659	HSBC 96 (CL)	09/11/95	11/08/96	0.1	88.4	30	2.574	Credit Lyonnais Fin (Guernsey) Ltd.
597	HSBC 96 (ML)	09/13/95	09/20/96	0.1	85	35	2.38	Merrill Lynch Int'l & Co. C. V.
534	HSBC 96 (UBS)	12/21/95	11/28/96	0.1	90	20	2.86	Union Bank of Switzerland
724	HSBC 97 (BT)	12/07/95	02/14/97	0.1	93.67	19	2.668	Bankers Trust Int'l plc
380	HSBC 97 (Peregn)	02/01/96	07/30/97	0.1	105	60	2.7	Peregrine Derivatives Ltd.
652	HSBC 97 (RF)	02/08/96	06/27/97	0.1	107.1	200	2.38	Robert Fleming & Co. Ltd.
324	HSBC 97 (SBC)	02/15/96	07/24/97	0.1	111.6	25	2.511	Swiss Bank Corp., HK
768	HSBC 97 (BT)	03/07/96	04/29/97	0.1	110	18	2.824	Bankers Trust Int'l plc
914	HSBC 97 (BZW)	02/07/96	02/27/97	0.1	104.125	55	2.692	BZW Warrants Ltd.
1709	HSBC 97 (ML)	06/12/96	07/21/97	0.1	105	50	2.08	Merrill Lynch Int'l & Co. C.V.
1719	HSBC 97 (UBS)	07/25/96	11/03/97	0.1	106.2	25	2.36	Union Bank of Switzerland
1771	HSBC 97 (BZW)	10/24/96	10/04/97	0.1	131.4	55	2.54	BZW Warrants Ltd.
1720	HSBC 97 (CL)	07/31/96	07/22/97	0.1	102.85	75	2.71	Credit Lyonnais F P (G) Ltd.
1755	HSBC 97 (ML)	10/03/96	10/06/97	0.1	146	120	1	Merrill Lynch Int'l & Co. C.V.
1788	HSBC 97 (RF)	11/07/96	10/15/97	0.1	143.9	100	2.065	Robert Fleming & Co. Ltd.
1741	HSBC 98 (BT)	09/11/96	02/16/98	0.1	112.2	20	3.221	Bankers Trust Int'l plc
1744	HSBC 98 (PARIBAS)	09/11/96	02/16/98	0.1	112.2	22.3	3.221	Paribas Capital Markets Group Ltd.
1880	HSBC 97 (BZW)	02/05/97	11/14/97	0.1	177.98	65	1.585	BZW Warrants Ltd.
1814	HSBC 97 (CARR)	12/11/96	11/26/97	0.1	146.7	40	2.869	Indosuez W.I. Carr (D) Ltd.
1821	HSBC 97 (CL)	12/12/96	11/16/97	0.1	184.2	180	0.998	Credit Lyonnais F P (G) Ltd.
1866	HSBC 97 (DMG)	01/29/97	11/21/97	0.1	176.4	54	1.68	Deutsche Bank AG
1928	HSBC 97 (DMG)	03/05/97	12/31/97	0.1	147	38	3.9988	Deutsche Bank AG
1812	HSBC 97 (ML)	12/05/96	11/20/97	0.1	176.5	230	1	Merrill Lynch Int'l & Co. C.V.
1920	HSBC 98 (ABN)	02/28/97	02/13/98	0.1	186	20	2.73	ABN AMRO Bank N.V.
1898	HSBC 97 (ML)	02/19/97	11/13/97	0.1	200	100	1.28	Merrill Lynch Int'l & Co. C.V.
1918	HSBC 98 (PARIBAS)	02/27/97	01/28/98	0.1	165.75	19	2.954	Paribas Capital Markets Group Ltd.
1857	HSBC 98 (UBS)	01/22/97	01/05/98	0.1	165	30	2.145	Union Bank of Switzerland
2019	HSBC 98 (BZW)	06/25/97	03/26/98	0.1	220.9	60	3.758	BZW Warrants Ltd.
1997	HSBC 98 (CARR)	06/04/97	06/04/98	0.1	231	25	2.244	Indosuez W.I. Carr (D) Ltd.
2035	HSBC 98 (ING)	06/25/97	05/29/98	0.1	227	27.5	3.1054	ING Baring Financial Products
2015	HSBC 98 (ML)	06/04/97	03/31/98	0.1	231	120	2.233	Merrill Lynch Int'l & Co. C.V.
2053	HSBC 98 (BEAR STEARNS)	08/06/97	05/15/98	0.1	292.95	28	3.2922	Bear Stearns Co. Inc.
2170	HSBC 98 (BZW)	09/25/97	06/29/98	0.1	236	40	3.766	BZW Warrants Ltd.
2085	HSBC 98 (CL)	08/06/97	05/07/98	0.1	300	56	2.6792	Credit Lyonnais F P (G) Ltd.
2022	HSBC 98 (ML)	08/06/97	05/18/98	0.1	279	120	4.096	Merrill Lynch Int'l & Co. C.V.
2127	HSBC 98 (ML)	09/12/97	06/12/98	0.1	299	100	2.3	Merrill Lynch Int'l & Co. C.V.
2058	HSBC 98 (MS)	07/23/97	06/04/98	0.1	253.05	25.5	2.9402	Morgan Stanley (Jersey) Ltd.
2067	HSBC 98 (MS)	08/07/97	06/17/98	0.1	292	19.5	3.5573	Morgan Stanley (Jersey) Ltd.
2157	HSBC 98 (NW)	09/25/97	06/30/98	0.1	212.4	13.3	5.0504	NatWest Financial Products plc
2078	HSBC 98 (SGA)	08/06/97	05/15/98	0.1	292.95	17	3.1527	SGA Societe Generale Acceptance N.V.
2152	HSBC 98 (SGA)	09/25/97	07/01/98	0.1	234.15	16	3.233	SGA Societe Generale Acceptance N.V.
2097	HSBC 98 (UBS)	08/06/97	05/21/98	0.1	275.1	28	2.963	Union Bank of Switzerland
2145	HSBC 98 (UBS)	09/24/97	06/29/98	0.1	260.4	35	3.5514	Union Bank of Switzerland
2191	HSBC 98 (ML)	11/26/97	09/04/98	0.1	182.5	40	3.722	Merrill Lynch Int'l & Co. C.V.
2163	HSBC 98 (SGA)	12/17/97	09/30/98	0.1	185.4	12	4.709	SGA Societe Generale Acceptance N.V.

Note: The data set consists of 59 HSBC derivative warrant series which were transacted on HKSE between September 1993 and December 1997. These derivative warrants were issued by 19 financial institutions. Information about warrants exercise prices, expire date, conversion ratios, dividends are compiled from various issues of the "South China Morning Post", the "Hong Kong Economic Journal Monthly". The units issued at initial listing, issue price and issue date are obtained from Stock Exchange Fact Books (HKSE, 1992 to 1997).

Table-2: The Black-Scholes model pricing errors.

Code	Warrant Name	Observations	Market Price (P_m)	Model Price (P_B)	Price Differences $[(P_m - P_B)/P_m]$	t-statistic
477	HSBC 94 (BZW)	58	5.081069	4.938667	0.026901	0.683947
589	HSBC 94 (Harvest)	42	4.23869	4.073907	0.035176	0.654109
653	HSBC 94 (Ford Deluxe)	133	3.949511	3.8499	0.023941	0.808428
1115	HSBC 95 (ML)	305	1.604049	1.523799	0.023299	1.97364**
573	HSBC 95 (Peregn)	307	3.639072	3.368218	0.060023	3.116692***
58	HSBC 95 (SBC)	451	2.348492	1.79742	0.197292	8.104017***
1116	HSBC 95 (SBC)	265	2.634509	2.469502	0.062579	3.179536***
768	HSBC 95 (Peregn)	330	2.316824	1.807122	0.150795	5.518742***
424	HSBC 96 (BZW)	459	0.669007	0.581314	0.022387	3.435418***
630	HSBC 96 (Peregn)	451	1.084645	0.873099	0.105409	6.244496***
682	HSBC 96 (R Flem)	471	0.546435	0.406078	0.030072	5.148959***
515	HSBC 96 (SBC)	202	2.858416	2.717695	0.052566	2.425914***
1226	HSBC 97 (MS)	306	4.238317	4.012871	0.054126	2.460266***
932	HSBC 97 (BZW)	230	3.042065	2.849367	0.065097	3.335983***
659	HSBC 96 (CL)	222	3.220721	3.109972	0.034893	1.969971**
597	HSBC 96 (ML)	124	3.432258	3.503997	-0.02161	-1.10819
534	HSBC 96 (UBS)	125	3.2394	3.137737	0.031172	1.157002
724	HSBC 97 (BT)	113	3.050442	2.889326	0.051897	2.207475**
380	HSBC 97 (Peregn)	170	3.617265	3.420213	0.076315	0.88945
652	HSBC 97 (RF)	280	4.246107	4.072172	0.070555	0.850299
324	HSBC 97 (SBC)	189	3.645106	3.516467	0.058797	0.592896
768	HSBC 97 (BT)	229	3.620568	3.567237	0.039846	0.268961
914	HSBC 97 (BZW)	126	2.31496	2.154341	0.071986	1.438167*
1709	HSBC 97 (ML)	80	3.656312	3.583752	0.032025	0.309844
1719	HSBC 97 (UBS)	173	6.288728	6.194674	0.024451	0.297006
1771	HSBC 97 (BZW)	164	6.024238	5.895715	0.035023	0.397773
1720	HSBC 97 (CL)	28	3.817857	3.692586	0.036614	0.655596
1755	HSBC 97 (ML)	174	4.57704	4.250163	0.11518	1.064273
1788	HSBC 97 (RF)	149	5.066779	4.797005	0.081183	0.824632
1741	HSBC 98 (BT)	73	8.166863	7.568193	0.085456	1.335015*
1744	HSBC 98 (PARIBAS)	95	8.151053	7.847052	0.043192	0.825765
1880	HSBC 97 (BZW)	133	4.067143	3.743913	0.138302	1.072121
1814	HSBC 97 (CARR)	101	4.54505	4.267095	0.078367	0.958629
1821	HSBC 97 (CL)	194	3.483557	2.987101	0.257881	2.160349**
1866	HSBC 97 (DMG)	103	3.419029	3.018316	0.170503	1.386857*
1928	HSBC 97 (DMG)	23	5.667391	5.431701	0.049669	0.337
1812	HSBC 97 (ML)	183	3.523005	3.083978	0.216424	1.831025**
1920	HSBC 98 (ABN)	178	4.670112	4.175072	0.142286	1.956492**
1898	HSBC 97 (ML)	152	3.197822	2.634073	0.294644	2.440787***
1918	HSBC 98 (PARIBAS)	46	5.034783	4.578418	0.117591	0.927921
1857	HSBC 98 (UBS)	135	4.668704	4.235808	0.122837	1.485034**
2019	HSBC 98 (BZW)	126	3.591667	2.777323	0.24463	3.295074***
1997	HSBC 98 (CARR)	134	3.520634	2.590297	0.220379	4.710297***
2035	HSBC 98 (ING)	93	4.157043	3.158892	0.242381	4.059583***
2015	HSBC 98 (ML)	140	3.326643	2.22996	0.337339	5.395682***
2053	HSBC 98 (BEAR STEARNS)	67	1.538328	0.638413	0.485321	6.953143***
2170	HSBC 98 (BZW)	64	2.072266	1.639655	0.139737	2.139479**
2085	HSBC 98 (CL)	80	1.426212	0.473715	0.459081	7.78717***
2022	HSBC 98 (ML)	95	1.630684	0.776313	0.387872	6.202318***
2127	HSBC 98 (ML)	56	1.017554	0.434708	0.436437	6.19434***
2058	HSBC 98 (MS)	99	2.690556	1.729703	0.268081	4.884574***
2067	HSBC 98 (MS)	66	1.651667	0.796965	0.403371	6.247507***
2157	HSBC 98 (NW)	49	3.270408	2.699285	0.166375	1.863618**
2078	HSBC 98 (SGA)	80	1.477925	0.603965	0.479217	7.470942***
2152	HSBC 98 (SGA)	64	2.200234	1.710511	0.177631	2.434424***
2097	HSBC 98 (UBS)	65	2.0312	1.027859	0.320438	5.508449***
2145	HSBC 98 (UBS)	45	1.601889	1.203872	0.095877	2.087126**
2191	HSBC 98 (ML)	24	3.9125	3.716338	0.053212	1.48219*
2163	HSBC 98 (SGA)	9	3.458333	3.436645	0.007305	0.176538

Note: *** indicates significance at the 1% level, ** indicates significance at the 5% level, and * indicates significance at the 10% level.

Table-3: The percentage pricing errors by the Black-Scholes model.

	Maturity Range		Historical Volatility	Moneyness								
				E/S<=0.9			0.9<E/S<=1.1			E/S>1.1		
				MPE	MSPE	MAPE	MPE	MSPE	MAPE	MPE	MSPE	MAPE
Panel A: Aggregate Results		0.5	1 month	0.0251	0.00454	0.03866	0.06291	0.44607	0.36258	-0.83076	7.1581	1.29032
			3 months	0.02927	0.00395	0.03939	0.11098	0.11744	0.25427	-1.28486	5.8707	1.58047
			6 months	0.03167	0.00362	0.03861	0.10568	0.06132	0.1918	-0.53598	1.17795	0.69346
			1 year	0.0332	0.00385	0.03932	0.06676	0.08384	0.20782	-0.01265	0.19271	0.32408
	0.5	1	1 month	0.04435	0.00523	0.05211	0.20726	0.06168	0.21725	-0.60419	3.95844	0.97965
			3 months	0.04637	0.005	0.05233	0.19112	0.06315	0.22361	-0.21965	1.10065	0.67538
			6 months	0.04581	0.00512	0.05205	0.2205	0.06666	0.23487	0.13522	0.33907	0.46021
			1 year	0.04424	0.00553	0.0518	0.23003	0.07601	0.242	0.26382	0.22268	0.39883
	1	1.5	1 month	0.11287	0.01915	0.11556	0.22799	0.07301	0.23624	0.59229	0.40296	0.60077
			3 months	0.10626	0.01673	0.10799	0.20837	0.04985	0.20857	0.48599	0.29067	0.48723
			6 months	0.10155	0.01614	0.10312	0.20063	0.04831	0.20457	0.26404	0.12471	0.27646
			1 year	0.10002	0.01717	0.10276	0.16599	0.042	0.17632	-0.00937	0.04427	0.16085
	1.5		1 month	0.06094	0.01486	0.10138	0.20709	0.05183	0.20877	0.15267	0.10753	0.25818
			3 months	0.08049	0.01958	0.11338	0.22863	0.06439	0.22927	0.06939	0.03921	0.16022
			6 months	0.09567	0.02323	0.11932	0.29121	0.09321	0.29121	0.08363	0.02917	0.13977
			1 year	0.08488	0.02787	0.13101	0.22857	0.06376	0.22857	0.36411	0.14811	0.36506
Panel B: Before Financial Crisis		0.5	1 month	0.02672	0.00321	0.03414	0.25052	0.12469	0.28431	0.44937	0.40985	0.56024
			3 months	0.02801	0.00331	0.03537	0.20014	0.08774	0.23758	0.25721	0.24183	0.39234
			6 months	0.02674	0.00286	0.03369	0.10759	0.05444	0.17744	0.03846	0.07601	0.20662
			1 year	0.02592	0.00284	0.03331	-0.01651	0.06854	0.17526	-0.1388	0.20118	0.30523
	0.5	1	1 month	0.04418	0.00508	0.05124	0.22555	0.06552	0.22752	0.22293	0.12397	0.29318
			3 months	0.0461	0.00496	0.05203	0.21605	0.0581	0.22046	0.24885	0.18791	0.37493
			6 months	0.04483	0.00498	0.05117	0.22383	0.06158	0.22556	0.31762	0.18488	0.35581
			1 year	0.04273	0.00527	0.05041	0.21229	0.06555	0.22566	0.08065	0.1383	0.29576
	1	1.5	1 month	0.11287	0.01915	0.11556	0.22799	0.07301	0.23624	0.59229	0.40296	0.60077
			3 months	0.10626	0.01673	0.10799	0.20837	0.04985	0.20857	0.48599	0.29067	0.48723
			6 months	0.10155	0.01614	0.10312	0.20063	0.04831	0.20457	0.26404	0.12471	0.27646
			1 year	0.10002	0.01717	0.10276	0.16599	0.042	0.17632	-0.0094	0.04427	0.16085
	1.5		1 month	0.06094	0.01486	0.10138	0.20709	0.05183	0.20877	0.15267	0.10753	0.25818
			3 months	0.08049	0.01958	0.11338	0.22863	0.06439	0.22927	0.06939	0.03921	0.16022
			6 months	0.09567	0.02323	0.11932	0.29121	0.09321	0.29121	0.08363	0.02917	0.13977
			1 year	0.08488	0.02787	0.13101	0.22857	0.06376	0.22857	0.36411	0.14811	0.36506
Panel C: After Financial Crisis		0.5	1 month	0.01827	0.01017	0.05768	-0.42575	1.28315	0.56643	-1.70124	11.7469	1.78677
			3 months	0.03454	0.00666	0.0563	-0.12126	0.19478	0.29773	-2.33347	9.69832	2.3884
			6 months	0.05243	0.00682	0.0593	0.1007	0.07924	0.22922	-0.9266	1.92726	1.02451
			1 year	0.06383	0.00811	0.06459	0.28365	0.1237	0.29262	0.07314	0.18695	0.3369
	0.5	1	1 month	0.05503	0.01472	0.10751	0.12467	0.04433	0.17086	-1.30538	7.20911	1.5616
			3 months	0.06316	0.00768	0.07131	0.0785	0.08599	0.23782	-0.61682	1.87442	0.93008
			6 months	0.10801	0.01423	0.10801	0.20548	0.08961	0.2769	-0.01942	0.4698	0.5487
			1 year	0.14008	0.02185	0.14008	0.31018	0.12326	0.31582	0.4191	0.29422	0.48621
	1	1.5	1 month	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
			3 months	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
			6 months	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
			1 year	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
	1.5		1 month	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
			3 months	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
			6 months	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
			1 year	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN

Table-4: The linear regression correction of the Black-Scholes percentage pricing errors.

$$\text{Model 1}: \eta_t^* = a_0 + a_1\tau + a_2(X/S_t) + a_3 D_{fc}$$

$$\text{Model 2}: \eta_t^* = a_0 + a_1\tau + a_2(X/S_t) + a_3 D_{fc} + a_4\tau^2 + a_5(X/S_t)^2 + a_6\tau(X/S_t)$$

$$\text{Model 3}: \eta_t^* = a_0 + a_1\tau + a_2(X/S_t) + a_3 D_{fc} + a_4\tau^2 + a_5(X/S_t)^2 + a_6\tau(X/S_t) + a_7\sigma_t$$

$$\text{Model 4}: \eta_t^* = a_0 + a_1\tau + a_2(X/S_t) + a_3 D_{fc} + a_4\tau^2 + a_5(X/S_t)^2 + a_6\tau(X/S_t) + a_7\sigma_t + a_8\sigma_t^2$$

where η_t^* is the percentage pricing error of the Black-Scholes model; τ is the time-to-maturity; X/S_t is the moneyness; D_{fc} is the dummy variable with a value of 1 after the financial crisis; σ_t is the historical volatility (with four different measures). *t*-statistics are presented in the parentheses. The 1% and 5% critical values for the *t*-test are 2.57 and 1.96, respectively. The 1% critical values for the *F*-statistics for Model 1 to Model 4 are 3.8, 2.8, 2.7 and 2.6, respectively.

Historical Volatility	Model	Coefficient Estimates a0	a1	a2	a3	a4	a5	a6	a7	a8	Adjusted R2	F-Stat
1 month	Model 1	0.489	0.1911	-0.543	-0.7661						0.166117	607.0593
		(14.138)	(7.674)	(-14.759)	(-27.822)							
	Model 2	-1.585	-1.9596	5.3682	-0.5295	-0.3689	-3.9413	2.9077			0.34616	806.3469
		(-15.801)	(-22.626)	(28.651)	(-20.777)	(-8.9423)	(-43.636)	(36.964)				
	Model 3	-0.2077	-2.0971	3.6456	0.1778	0.2052	-2.6077	2.061	-2.6618		0.529638	1469.171
		(-2.3564)	(-28.534)	(22.571)	(7.2111)	(5.6547)	(-32.678)	(30.215)	(-59.656)			
	Model 4	-1.4076	-1.5253	3.7833	0.0029	0.0629	-2.4673	1.545	2.471	-4.2867	0.627672	1924.293
		(-17.132)	(-22.965)	(26.322)	(0.1309)	(1.9413)	(-34.722)	(25.084)	(22.064)	(-49.01)		
3 months	Model 1	0.2876	0.1544	-0.3065	-0.6342						0.194324	734.7929
		(11.852)	(8.8403)	(-11.874)	(-32.833)							
	Model 2	-0.5973	-1.0822	2.4868	-0.4643	-0.3727	-2.1071	1.9741			0.320253	717.6754
		(-8.1821)	(-17.171)	(18.239)	(-25.035)	(-12.417)	(-32.059)	(34.486)				
	Model 3	0.0734	-1.3321	2.0145	-0.0282	-0.0272	-1.6418	1.6727	-1.8868		0.410899	910.4433
		(1.0451)	(-22.558)	(15.793)	(-1.3567)	(-0.9258)	(-26.295)	(31.038)	(-37.476)			
	Model 4	-1.132	-0.9076	1.9284	0.0253	-0.0501	-1.3944	1.1504	4.8294	-7.9803	0.51949	1234.424
		(-16.456)	(-16.763)	(16.738)	(1.3452)	(-1.8854)	(-24.614)	(23.002)	(31.211)	(-45.41)		
6 months	Model 1	0.0264	0.1101	0.0032	-0.1929						0.078755	261.0831
		(1.9938)	(11.546)	(0.2239)	(-18.289)							
	Model 2	-0.4365	-0.1443	1.1621	-0.1225	-0.2656	-0.8489	0.7592			0.160044	290.8417
		(-10.535)	(-4.033)	(15.016)	(-11.639)	(-15.59)	(-22.755)	(23.365)				
	Model 3	-0.1086	-0.3185	1.0444	0.0177	-0.1116	-0.6953	0.6966	-1.09		0.246638	427.8616
		(-2.679)	(-9.2844)	(14.232)	(1.6243)	(-6.6314)	(-19.503)	(22.592)	(-32.394)			
	Model 4	-1.0152	-0.1707	0.8059	0.0851	-0.1644	-0.5679	0.6158	5.3621	-9.0573	0.388634	726.2337
		(-24.475)	(-5.4941)	(12.153)	(8.5912)	(-10.82)	(-17.619)	(22.128)	(37.394)	(-46.04)		
1 year	Model 1	-0.1001	0.1539	0.0651	0.2149						0.169216	620.6697
		(-11.928)	(25.473)	(7.289)	(32.17)							
	Model 2	-0.2753	0.1192	0.457	0.2404	-0.1206	-0.2887	0.2611			0.193421	365.781
		(-10.165)	(5.1)	(9.0347)	(34.938)	(-10.826)	(-11.838)	(12.295)				
	Model 3	-0.1175	0.0657	0.4662	0.2576	-0.0563	-0.2241	0.2145	-0.7081		0.246889	428.437
		(-4.3681)	(2.8937)	(9.5373)	(38.544)	(-5.0947)	(-9.4565)	(10.414)	(-25.467)			
	Model 4	-0.3569	0.0702	0.4403	0.2609	-0.08	-0.2224	0.2479	1.0851	-2.9177	0.252942	387.2829
		(-9.2689)	(3.106)	(9.0279)	(39.136)	(-7.0551)	(-9.4246)	(11.873)	(5.1909)	(-8.655)		

Table-5: The mean squared percentage pricing error ratios (the linear regression corrected model vs. the Black-Scholes model).

For the across-warrant analysis, the sample is divided into three groups: 30 warrant series in S1, 14 in S2, and 15 in S3. For the across-time analysis, the sample is divided into three subperiods: T1 (from Sep. 1993 to Dec. 1995), T2 (from Jan. 1996 to Mar. 1997), and T3 (after Mar. 1997). This table presents the mean squared percentage error (MSPE) ratio, which is the MSPE of the linear regression corrected model divided by the MSPE of the Black-Scholes model. The results corresponding to S1+S2 or T1+T2 are based on the in-sample analysis, whereas the results for S3 or T3 are based on the out-of-sample analysis. Each of four measures for the historical volatility (1 month, 3 months, 6 months and 1 year) is used to obtain the results.

		Entire Data Set					Only Observations Before Financial Crisis				
		Obs.	1 Month	3 Months	6 Months	1 Year	Obs.	1 Month	3 Months	6 Months	1 Year
Across warrants											
Ratio of in-sample MSPEs (S1+S2)	Total	6650	0.137565	0.180468	0.434443	0.817035	5599	0.913395	1.043424	0.760511	0.756146
	Before Financial Crisis	5599	1.376994	1.084723	0.88864	0.820991	5599	0.913395	1.043424	0.760511	0.756146
	In the Money	4053	2.386064	1.884995	1.118634	0.777089	4053	0.729504	0.548094	0.545029	0.696176
	At the Money	337	0.844193	0.666836	0.513941	0.559043	337	0.510406	0.559847	0.474502	0.535547
	Out of the Money	1209	1.21959	0.94309	0.861433	0.883747	1209	0.982558	1.198303	0.886809	0.816787
	Time to Maturity<=0.75	2517	1.418838	1.365215	1.167508	0.797945	2517	1.116497	1.090976	1.099682	0.7151
	Time to Maturity>0.75	3082	1.351836	0.954128	0.793227	0.837068	3082	0.791365	1.021296	0.644468	0.784778
	After Financial Crisis	1051	0.029561	0.069631	0.259606	0.811841					
	In the Money	282	2.674663	8.375995	3.517152	2.246237					
	At the Money	85	0.429332	0.93336	0.521688	0.29751					
	Out of the Money	684	0.021221	0.055627	0.227383	0.782646					
	Time to Maturity<=0.75	971	0.028826	0.068271	0.25689	0.833099					
	Time to Maturity>0.75	80	0.680486	0.56958	0.412502	0.564572					
Ratio of out-sample MSPEs (S3)	Total	2478	0.072961	0.150229	0.414522	0.767657	2038	0.619551	0.621342	0.581408	0.742677
	Before Financial Crisis	2038	1.181787	0.880014	0.672408	0.757097	2038	0.619551	0.621342	0.581408	0.742677
	In the Money	1247	2.300826	1.776425	0.93745	0.75323	1247	0.698216	0.603672	0.641059	0.722361
	At the Money	245	0.67931	0.705371	0.548321	0.595101	245	0.388762	0.497447	0.482969	0.556593
	Out of the Money	546	0.984218	0.652132	0.616104	0.797898	546	0.660365	0.65016	0.58463	0.795498
	Time to Maturity<=0.75	925	1.578983	1.298929	0.755258	1.000617	925	0.566384	0.75532	0.687527	1.038999
	Time to Maturity>0.75	1113	1.063154	0.735062	0.64456	0.690821	1113	0.635428	0.574985	0.54574	0.662033
	After Financial Crisis	440	0.025567	0.077908	0.307868	0.777942					
	In the Money	137	4.265618	1.923032	1.453377	2.33594					
	At the Money	49	0.508397	1.318052	1.560246	0.596827					
	Out of the Money	254	0.021992	0.061812	0.272339	0.720566					
	Time to Maturity<=0.75	433	0.02548	0.076875	0.307021	0.773216					
	Time to Maturity>0.75	7	0.283287	0.991831	1.320355	5.661759					
Across time periods											
Ratio of in-sample MSPEs (T1+T2)	Total	6656	0.856352	0.985648	0.752807	0.755847	6656	0.856352	0.985648	0.752807	0.755847
	Before Financial Crisis	6656	0.856352	0.985648	0.752807	0.755847	6656	0.856352	0.985648	0.752807	0.755847
	In the Money	4498	0.678967	0.535628	0.534532	0.658648	4498	0.678967	0.535628	0.534532	0.658648
	At the Money	505	0.45437	0.555547	0.485248	0.591828	505	0.45437	0.555547	0.485248	0.591828
	Out of the Money	1653	0.941373	1.134445	0.874057	0.818741	1653	0.941373	1.134445	0.874057	0.818741
	Time to Maturity<=0.75	2662	1.097112	1.111283	1.152197	0.800887	2662	1.097112	1.111283	1.152197	0.800887
	Time to Maturity>0.75	3994	0.739311	0.934288	0.630674	0.732837	3994	0.739311	0.934288	0.630674	0.732837
	After Financial Crisis										
	In the Money										
	At the Money										
	Out of the Money										
	Time to Maturity<=0.75										
	Time to Maturity>0.75										
Ratio of out-sample MSPEs (T3)	Total	2472	1.521964	3.191993	2.511752	0.93282	981	0.574123	0.304016	0.382026	0.637293
	Before Financial Crisis	981	0.574123	0.304016	0.382026	0.637293	981	0.574123	0.304016	0.382026	0.637293
	In the Money	802	1.367735	0.591844	0.467788	0.435782	802	1.367735	0.591844	0.467788	0.435782
	At the Money	77	0.083729	0.130212	0.242303	0.56492	77	0.083729	0.130212	0.242303	0.56492
	Out of the Money	102	0.443669	0.230048	0.391572	0.746482	102	0.443669	0.230048	0.391572	0.746482
	Time to Maturity<=0.75	780	0.554466	0.379113	0.437644	0.700556	780	0.554466	0.379113	0.437644	0.700556
	Time to Maturity>0.75	201	0.614328	0.220716	0.33237	0.576199	201	0.614328	0.220716	0.33237	0.576199
	After Financial Crisis	1491	1.524129	3.208678	2.581257	0.974514					
	In the Money	419	0.460565	0.833844	0.784609	0.877818					
	At the Money	134	0.346871	0.819412	0.674994	0.932201					
	Out of the Money	938	1.529299	3.222581	2.627731	0.982015					
	Time to Maturity<=0.75	1404	1.524883	3.214532	2.607029	0.984665					
	Time to Maturity>0.75	87	0.587758	0.591151	0.56491	0.790354					

Table-6: The bandwidth selection for the LLKR correction.

The optimal bandwidths based on the cross-validation method are reported for different regressors under different scenarios. h1 is the bandwidth for time to maturity, h2 is the bandwidth for moneyness and h3 is the bandwidth for historical volatility.

		Historical Volatility	h1	h2	h3
Entire Data Set	**Across warrants**	*1 month*	0.616749	0.125778	0.006567
		3 months	0.670924	0.147863	0.001285
		6 months	0.418398	0.184017	0.002793
		1 year	0.616353	0.069566	0.04161
	Across time periods	*1 month*	0.886653	0.13241	0.138289
		3 months	0.541713	0.077173	0.330192
		6 months	0.230823	0.077213	0.093231
		1 year	0.461584	0.033091	0.104281
Before Financial Crisis	**Across warrants**	*1 month*	0.389025	0.072561	0.026362
		3 months	0.630932	0.13393	0.005668
		6 months	1.009424	0.089275	0.040345
		1 year	0.326865	0.050223	0.13675
	Across time periods	*1 month*	0.886653	0.13241	0.138289
		3 months	0.541713	0.077173	0.330192
		6 months	0.230823	0.077213	0.093231
		1 year	0.461584	0.033091	0.104281

Table-7: The mean squared percentage pricing error ratios (the LLKR corrected model vs. the Black-Scholes model).

For the across-warrant analysis, the sample is divided into three groups: 30 warrant series in S1, 14 in S2, and 15 in S3. For the across-time analysis, the sample is divided into three subperiods: T1 (from Sep. 1993 to Dec. 1995), T2 (from Jan. 1996 to Mar. 1997), and T3 (after Mar. 1997). This table presents the mean squared percentage error (MSPE) ratio, which is the MSPE of the local linear kernel regression corrected model divided by the MSPE of the Black-Scholes model. The results corresponding to S1+S2 or T1+T2 are based on the in-sample analysis, whereas the results for S3 or T3 are based on the out-of-sample analysis. Each of four measures for the historical volatility (1 month, 3 months, 6 months and 1 year) is used to obtain the results.

		Entire Data Set					Only Observations Before Financial Crisis				
		Obs.	1 Month	3 Months	6 Months	1 Year	Obs.	1 Month	3 Months	6 Months	1 Year
Across warrants											
Ratio of in-sample MSPEs (S1+S2)	Total	6650	0.041596	0.028156	0.167815	0.482842	5599	0.411207	0.198242	0.325042	0.291643
	Before Financial Crisis	5599	0.432871	0.161954	0.436814	0.403343	5599	0.411207	0.198242	0.325042	0.291643
	In the Money	4053	0.318316	0.193473	0.222784	0.438698	4053	0.290333	0.225963	0.286386	0.396558
	At the Money	337	0.380028	0.443314	0.438077	0.398609	337	0.350155	0.569284	0.368751	0.312968
	Out of the Money	1209	0.459911	0.129417	0.517103	0.392243	1209	0.440168	0.158439	0.332657	0.252218
	Time to Maturity<=0.75	2517	0.933551	0.326477	0.643677	0.398569	2517	0.891595	0.389086	0.609222	0.33415
	Time to Maturity>0.75	3082	0.132069	0.085346	0.366038	0.406675	3082	0.122597	0.109378	0.227817	0.261994
	After Financial Crisis	1051	0.0075	0.011756	0.064268	0.587197					
	In the Money	282	0.158392	0.367215	0.34054	0.394478					
	At the Money	85	0.171962	0.161278	0.294823	0.517321					
	Out of the Money	684	0.006669	0.010899	0.058545	0.602065					
	Time to Maturity<=0.75	971	0.007159	0.011214	0.062633	0.547983					
	Time to Maturity>0.75	80	0.309383	0.211238	0.1563	1.043273					
Ratio of out-sample MSPEs (S3)	Total	2478	0.016905	0.039222	0.160113	0.544913	2038	0.361839	0.299799	0.403888	0.552567
	Before Financial Crisis	2038	0.290947	0.282975	0.337999	0.64083	2038	0.361839	0.299799	0.403888	0.552567
	In the Money	1247	0.491303	0.429531	0.436157	0.541614	1247	0.740446	0.415557	0.385547	0.551353
	At the Money	245	0.341874	0.280852	0.303055	0.583828	245	0.322798	0.324666	0.324313	0.565107
	Out of the Money	546	0.216145	0.240672	0.314665	0.691877	546	0.258345	0.261317	0.427657	0.549969
	Time to Maturity<=0.75	925	0.503532	0.580084	0.617973	0.846423	925	0.507551	0.627899	0.645829	0.959814
	Time to Maturity>0.75	1113	0.227454	0.180166	0.243895	0.584876	1113	0.318319	0.186266	0.322568	0.44174
	After Financial Crisis	440	0.005192	0.015067	0.086545	0.451505					
	In the Money	137	0.490342	0.155492	0.496	0.498623					
	At the Money	49	0.216861	0.304504	0.568304	0.760782					
	Out of the Money	254	0.004307	0.012677	0.073345	0.432404					
	Time to Maturity<=0.75	433	0.005176	0.014878	0.085468	0.451381					
	Time to Maturity>0.75	7	0.05362	0.181854	1.373792	0.839675					
Across time periods											
Ratio of in-sample MSPEs (T1+T2)	Total	6656	0.565741	0.36251	0.328896	0.375585	6656	0.565741	0.36251	0.328896	0.375585
	Before Financial Crisis	6656	0.565741	0.36251	0.328896	0.375585	6656	0.565741	0.36251	0.328896	0.375585
	In the Money	4498	0.402153	0.339227	0.293843	0.439988	4498	0.402153	0.339227	0.293843	0.439988
	At the Money	505	0.411782	0.571266	0.370596	0.474717	505	0.411782	0.571266	0.370596	0.474717
	Out of the Money	1653	0.618951	0.3461	0.334299	0.33565	1653	0.618951	0.3461	0.334299	0.33565
	Time to Maturity<=0.75	2662	0.549304	0.597721	0.664404	0.479613	2662	0.549304	0.597721	0.664404	0.479613
	Time to Maturity>0.75	3994	0.573725	0.266369	0.226299	0.322441	3994	0.573725	0.266369	0.226299	0.322441
	After Financial Crisis										
	In the Money										
	At the Money										
	Out of the Money										
	Time to Maturity<=0.75										
	Time to Maturity>0.75										
Ratio of out-sample MSPEs (T3)	Total	2472	0.027122	0.06258	0.593094	0.813355	981	0.379826	0.123425	0.142342	0.185139
	Before Financial Crisis	981	0.379826	0.123425	0.142342	0.185139	981	0.379826	0.123425	0.142342	0.185139
	In the Money	802	1.015844	0.260652	0.261735	0.263573	802	1.015844	0.260652	0.261735	0.263573
	At the Money	77	0.183412	0.078573	0.091325	0.148392	77	0.183412	0.078573	0.091325	0.148392
	Out of the Money	102	0.190735	0.073644	0.106623	0.165122	102	0.190735	0.073644	0.106623	0.165122
	Time to Maturity<=0.75	780	0.294856	0.126856	0.170319	0.222347	780	0.294856	0.126856	0.170319	0.222347
	Time to Maturity>0.75	201	0.553646	0.119619	0.117366	0.149203	201	0.553646	0.119619	0.117366	0.149203
	After Financial Crisis	1491	0.026316	0.062228	0.607804	0.901987					
	In the Money	419	0.429735	1.673726	0.916324	0.672592					
	At the Money	134	0.397431	1.395995	0.96872	0.804458					
	Out of the Money	938	0.024561	0.053772	0.599338	0.919588					
	Time to Maturity<=0.75	1404	0.025908	0.059177	0.604149	0.929872					
	Time to Maturity>0.75	87	0.532832	1.426485	0.893756	0.396167					

Table-8: Testing the effect of the issuers' identity.

The percentage pricing errors of the Black-Schloes model and those of the local linear kernel regression corrected model are regressed on the identity of the issuer. The observations before the Asian Financial Crisis are used for the analysis, and during this period there were 18 financial institutions issuing the HSBC derivative warrants. The regression model takes the form: $\eta^* = a_0 + \sum_i a_i D_i$ where η^* is the percentage pricing errors of the model and D_i is a dummy variable that equals 1 when the warrant is issued by institution *i* and 0 otherwise. The historical volatilities are calculated using the preceding 3 months' daily returns. *t*-statistics are presented in the parentheses.

	Dependent Variables					
	Percentage Pricing Errors of the Black-Scholes Model		**Percentage Pricing Errors of the LLKR Adjusted Model (based on the across warrants bandwidth selection)**		**Percentage Pricing Errors of the LLKR Adjusted Model (based on the across time periods bandwidth selection)**	
a0	0.00622	*(0.0827)*	0.00128	*(0.0212)*	-0.0153	*(-0.298)*
a1(Barclays de Zoete Wedd Warrants Ltd.)	0.16734	*(2.2186)*	-0.012	*(-0.198)*	-0.0147	*(-0.285)*
a2(Harvest Top Investment Ltd.)	0.02784	*(0.3428)*	0.02057	*(0.3146)*	0.03563	*(0.6406)*
a3(Ford Deluxe Investment Ltd.)	0.01271	*(0.1647)*	-0.0044	*(-0.071)*	0.00619	*(0.1171)*
a4(Merrill Lynch International & Co. C.V.)	0.138	*(1.8286)*	-0.0161	*(-0.264)*	0.00959	*(0.1855)*
a5(Peregrine Derivatives Ltd.)	0.17092	*(2.2665)*	0.03952	*(0.651)*	0.02072	*(0.4012)*
a6(Swiss Bank Corp., HK)	0.14577	*(1.9322)*	0.03448	*(0.5677)*	0.04186	*(0.8101)*
a7(Robert Fleming & Co. Ltd.)	0.26194	*(3.4696)*	0.00094	*(0.0155)*	0.00714	*(0.1381)*
a8(Morgan Stanley (Jersey) Ltd.)	0.05927	*(0.7799)*	0.0112	*(0.1831)*	0.00157	*(0.0301)*
a9(Credit Lyonnais Fin (Guernsey) Ltd.)	0.113	*(1.4899)*	-0.0048	*(-0.079)*	0.02697	*(0.5192)*
a10(Union Bank of Switzerland)	0.04477	*(0.59)*	-0.0198	*(-0.325)*	0.00038	*(0.0074)*
a11(Bankers Trust Int'l plc)	0.07355	*(0.9698)*	-0.0268	*(-0.439)*	-0.0021	*(-0.039)*
a12(Paribas Capital Markets Group Ltd.)	0.04878	*(0.6269)*	0.00446	*(0.0712)*	0.01633	*(0.3064)*
a13(Indosuez W.I. Carr (D) Ltd.)	0.08552	*(1.1099)*	-0.0362	*(-0.583)*	0.01404	*(0.266)*
a14(Deutsche Bank AG)	0.11159	*(1.4403)*	-0.0433	*(-0.694)*	-0.0192	*(-0.362)*
a15(ABN AMRO Bank N.V.)	0.11513	*(1.4775)*	-0.0313	*(-0.499)*	0.00812	*(0.1521)*
a16(ING Baring Financial Products)	0.06128	*(0.736)*	-0.05	*(-0.746)*	0.00406	*(0.0712)*
a17(Bear Stearns Co. Inc.)	0.01522	*(0.1431)*	0.0152	*(0.1775)*	0.0152	*(0.2087)*
R2	0.08156		0.0213		0.01417	
F statistic	40.8881		10.7737		7.4561	

Figure-1. The HSBC Price and Return Volatility (based on the preceding year's daily returns)

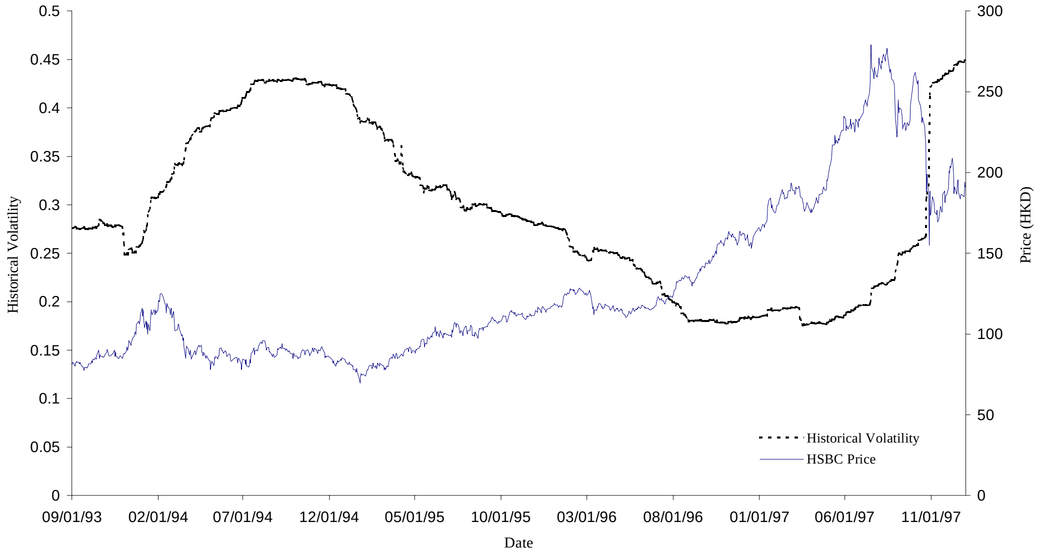
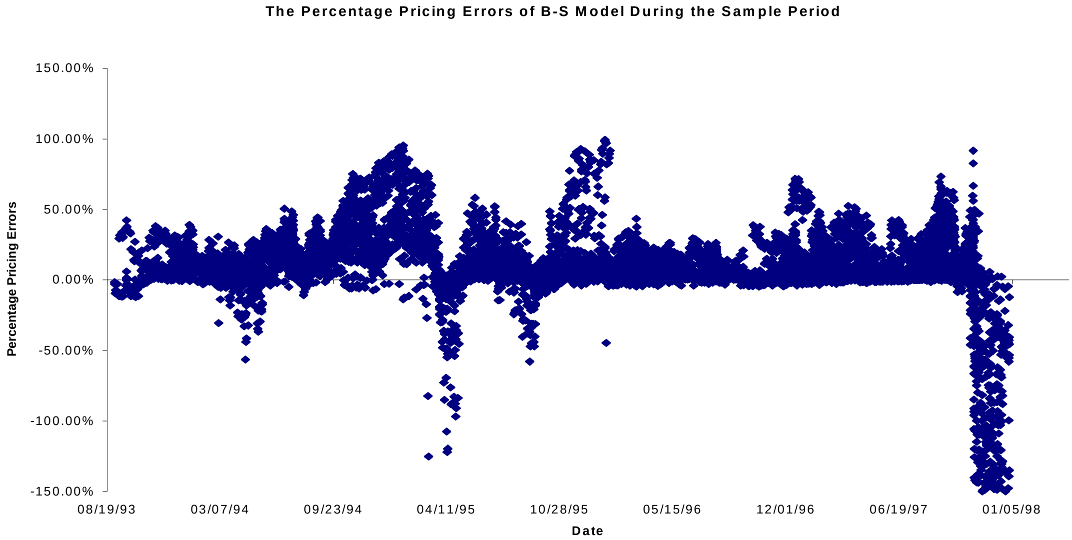
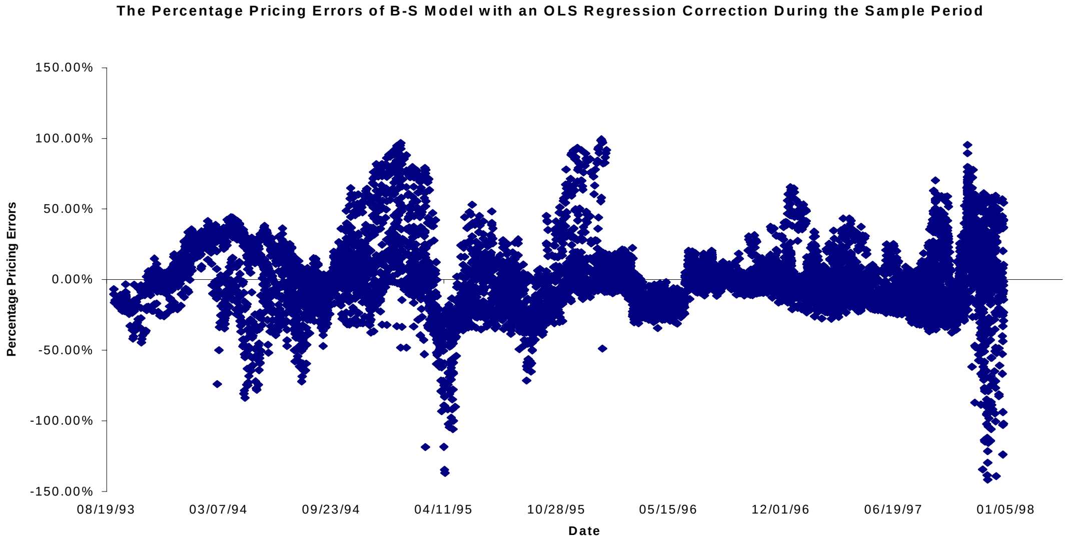
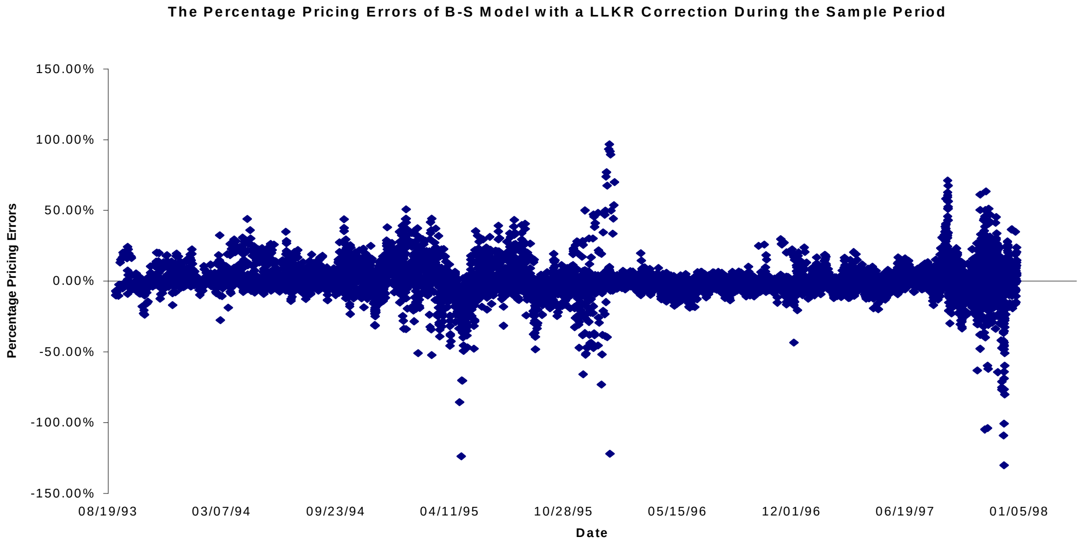


Figure-2. The Percentage Pricing Errors of B-S Model, B-S Model with an OLS Regression Correction and B-S Model with a LLKR Correction







Note: The historical volatilities in B-S model are calculated based on previous three monthes' daily stock returns.

Figure-3. The percentage pricing errors for various percentiles (5%, 25%, 50%, 75% and 95%) in the out-of-sample (across different warrant series) analysis based on different pricing methods.

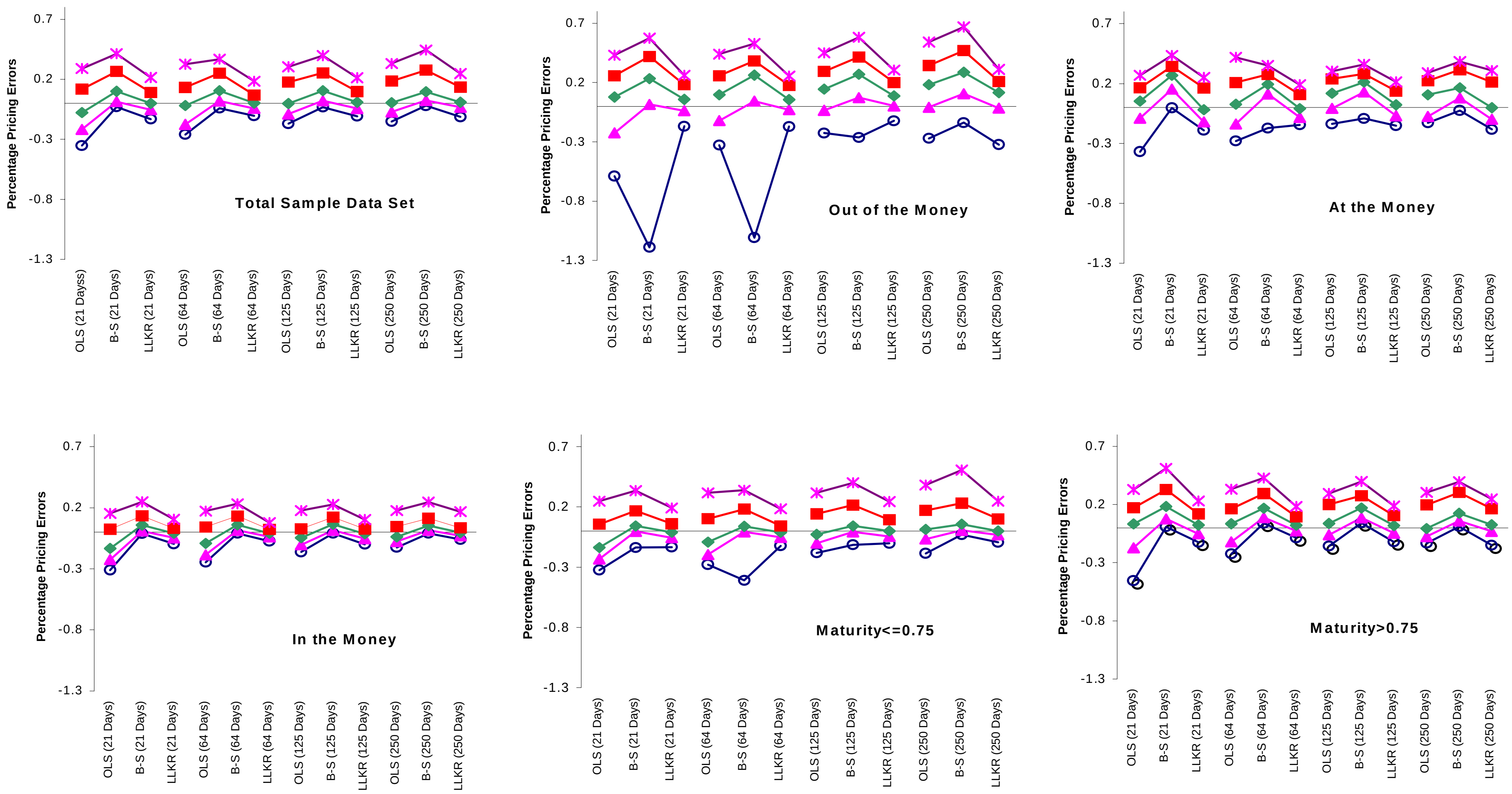


Figure-4. The percentage pricing errors for various percentiles (5%, 25%, 50%, 75% and 95%) in the out-of-sample (across different time periods) analysis based on different pricing methods.