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Probability of Default implied Rating (PDiR2.0)

White Paper

The Credit Research Initiative (CRI)
National University of Singapore

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ABSTRACT

First introduced by the Credit Research Initiative (CRI) in 2011, the Probability of Default Implied Rating (PDiR) complements the CRI one-year Probability of Default (PD) by providing a convenient and intuitive overview on the credit quality of a firm through mapping the CRI PD to letter grades used by major rating agencies. The PDiR2.0 methodology has been developed to enhance the PD mapping by targeting the average realized credit rating migration experienced by the S&P or Moody's global corporate rating pool instead of relying solely on the reported default rates of the pool. This white paper seeks to provide a more intuitive explanation of the **PDiR2.0** by a methodological walkthrough complemented with an illustrative example. Users are referred to the scientific paper, "Enhanced PD-implied Ratings by Targeting the Credit Rating Migration Matrix" by Duan and Li (2021), for technical details.

CONTENT

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I. OVERVIEW

Credit ratings in the form of alphabetical letter-grades are intended to give users a convenient and intuitive overview of an obligor's creditworthiness. The CRI Probability of Default Implied Rating (PDiR) was first introduced in 2011 to complement the high-granularity CRI Probability of Default (CRI-PD) by assigning a letter-grade to each firm according to a systematic mapping of one-year PD based on observed historical default rates from the Standard & Poor's (S&P) credit ratings.

The methodology was later revised and implemented on December 15, 2017 to provide a better match between expected default rates predicted by the CRI PD and the average historical default rates of the S&P or Moody's global corporate rating pool. Hereafter, we will refer to this methodology as $PDiR_{old}$.[1]

Due to the lack of realized defaults for AAA and AA+ for the S&P rating pool, $PDiR_{old}$ relies on proxy values from a linear extrapolation for these two rating cohorts. These extrapolated values are arguably arbitrary because letter ratings are ordinal rankings without a suitable cardinal interpretation. Moreover, a recent effort to tally the proportion of the firms in the CRI sample falling into each of the rating categories suggests that there have been too many firms in the AAA category as compared to the experience of the S&P or Moody's global corporate rating pool. These two considerations have led to the revision effort to roll out PDiR2.0.

The PDiR2.0 methodology developed in 2020 and published in Duan and Li (2021) targets the historical credit rating migration experience of the S&P or Moody's global corporate rating pool instead of relying solely on the reported default rates of the pool. PDiR2.0 has been implemented by the CRI team starting April 13, 2020. The PD cutoff values defining different rating categories will be updated occasionally to incorporate significant changes in the corporate credit markets.

Coverage

The CRI provides PDiR2.0 for every public firm under its PD coverage. Correspondingly, this means PDiR2.0 covers the same sample as the CRI PD, on over 85,000 exchange-listed

[1] Duan and Li (2020) provides a scientific foundation for the $PDiR_{old}$ methodology.

firms in 133 economies around the world over roughly a 30-year period. PDiR2.0 are also updated daily for over 43,000 currently active exchange-listed firms in the CRI database.

II. THE MAPPING TABLE

Table 1 presents the CRI PDiR2.0 mapping table as of April 13, 2020, calibrated to the realized credit rating migration history of the S&P global corporate rating pool over an 18-year period (2000-2017).

Table 1. Mapping 10-day moving average 1-year CRI PD to the S&P experience

Rating Category	Initial Assignment		Upgrade To		Downgrade To	
	lb (bps)	ub (bps)	lb (bps)	ub (bps)	lb (bps)	ub (bps)
AAA	0	0.0035	0	0.0027	-	-
AA+	0.0035	0.1044	0.0027	0.0035	0.1044	0.3060
AA	0.1044	0.3060	0.0035	0.1044	0.3060	0.4069
AA-	0.3060	0.4069	0.1044	0.3060	0.4069	1.2928
A+	0.4069	1.2928	0.3060	0.4069	1.2928	3.0646
A	1.2928	3.0646	0.4069	1.2928	3.0646	3.9506
A-	3.0646	3.9506	1.2928	3.0646	3.9506	9.9936
BBB+	3.9506	9.9936	3.0646	3.9506	9.9936	22.0796
BBB	9.9936	22.0796	3.9506	9.9936	22.0796	28.1227
BBB-	22.0796	28.1227	9.9936	22.0796	28.1227	46.2056
BB+	28.1227	46.2056	22.0796	28.1227	46.2056	82.3715
BB	46.2056	82.3715	28.1227	46.2056	82.3715	100.4544
BB-	82.3715	100.4544	46.2056	82.3715	100.4544	357.0556
B+	100.4544	357.0556	82.3715	100.4544	357.0556	870.2578
B	357.0556	870.2578	100.4544	357.0556	870.2578	1126.8589
B-	870.2578	1126.8589	357.0556	870.2578	1126.8589	1630.8764
CCC+	1126.8589	1630.8764	870.2578	1126.8589	1630.8764	2638.9113
CCC	1630.8764	2638.9113	1126.8589	1630.8764	2638.9113	3142.9287
CCC-	2638.9113	3142.9287	1630.8764	2638.9113	3142.9287	4449.8571
CC	3142.9287	8370.6423	2638.9113	7063.7139	4449.8571	8777.9817
C	8370.6423	10000	-	-	8777.9817	10000

A similar mapping table calibrated to Moody's global rating pool over 19 years (2000-2018) is provided at the end of this paper (Table 3).

Rating Assignment Using the Mapping Table

- Apply 10-business day moving average PDs to map against the cutoff values. The use of moving average PDs helps minimize frequent rating changes due to oscillation across a boundary.

- **Initial Assignment**: Obtain initial ratings for any firms by mapping the moving average against the upper and lower bounds in the first two columns of, say, Table 1.

- **Upgrade/Downgrade**: The upper and lower bounds for upgrade/downgrade to a specific cohort are defined in the last 4 columns of the same table. The design with migration buffer zones creates latency in rating changes, intending to mimic the commercial credit rating practice. For example, a firm is upgraded from A+ to AA only if its moving average PD is smaller than AA's lower bound defined in the initial assignment.

III. METHODOLOGY

Mapping the CRI one-year PD to its corresponding PDiR2.0 requires the upper and lower bounds for all rating cohorts. If these PD cutoff values are in place, one can track rating migrations from the beginning to the end for each one-year period for the firms in the CRI database. But of course, they are not. We thus need an algorithmic way to find these cutoff values with a goal of best matching the one-year realized credit migration rates of the CRI sample with the historical average migration rates of a corporate rating pool.

For the remainder of this paper, we use the S&P global corporate rating pool to describe the methodology and the results. In the following, we briefly sketch the PDiR2.0 methodology developed in Duan and Li (2021).

The Observed Rating Migration Matrix

We obtain the S&P average realized one-year migration matrix over the 18-year sample period (from 2000 to 2017). Our data source is European Securities and Markets Authority (ESMA)'s central repository (CEREP). The ESMA database makes available the 9 consolidated rating cohorts from the original scale of 21 rating categories by lumping together those with a plus/minus modifier.

A firm with a rating at the beginning of a period may disappear at the end of the period for at least two competing reasons. One reason is bankruptcy/default for which both CRI and S&P track. The other form of exit is a withdrawal from rating service provided by S&P, which can be due to several reasons. However, it is reasonable to conjecture that a lowly rated firm, say CC, likely choose to forgo credit rating because it makes little sense to pay for a credit rating that explicitly reveals its poor credit quality. The CRI sample, on the other hand, faces the other form of exit arising from de-listings, which occur primarily due to mergers/acquisitions.

We can therefore anticipate substantially different other-exit rates facing S&P and CRI because their exit reasons differ. Technically, one must account for this difference to avoid unintended distortions. PDiR2.0 addresses this by grossing up each row of the migration matrix with the one minus other-exit rate that is unique to each rating cohort. The grossed-up rating migration matrix is 9 by 10 with the last column holding the default rates corresponding to the 9 rating cohorts. The adjusted migration matrix naturally has its row sums all equal to 1.

The average realized migration rates for the S&P pool over the 18-year sample period is computed by (1) summing up the 18 one-year migration matrices which records the total counts of transition to the 10 columns mentioned above and (2) dividing each entry by its row sum. This is equivalent to grossing up different rows by their respective S&P withdrawal rates. Denote our target, this grossed-up average realized rating migration matrix, by $\widehat{\boldsymbol{M}}$.

The Model's Implied Migration Matrix

The upper PD bound for a rating category simultaneously serves as the lower bound of the adjacent category of less credit quality. Therefore, the 8 cutoff values along with two

natural PD bounds of 0 and 1 define the 9 consolidated rating categories. These 8 PD upper bounds are denoted by $\boldsymbol{\theta} = (U_{AAA}, U_{AA}, U_A, U_{BBB}, U_{BB}, U_B, U_{CCC}, U_{CC})$, and they must be increasing in values. We also need to define cutoff values for ratings with plus and minus modifiers, and the 21 finer rating categories in turn help set migration buffer zones to generate rating stickiness exhibited by commercial credit ratings. Without these buffer zones, ratings are likely to flip back forth for firms with their PDs close to a boundary value.

Each of the PD segment (a lower bound to an upper bound is equally divided into four subsegments. The top (bottom) 25% subsegment (measured as the percentage of PDs) is reserved for the rating with a minus (plus) modifier. Initial rating assignments are based on the 21 categories so defined. Migration to a rating category only occurs if its 10-day moving average PD crosses beyond a complete finer rating category; that is, an A obligor is downgraded to BBB+ only if its PD moves into the interval defined by BBB. Likewise, to upgrade a BBB+ obligor to A, its PD must move into the interval defined by A+. Category AAA, CC and C do not carry a rating modifier, but they still need buffer zones for migration assignments. To upgrade a firm to AAA (or CC), the 10-day moving average PD must be lower than the PD level corresponding to 75% of the AAA (or CC) interval. To downgrade a firm to CC (or C), the same logic applies, but is instead at the 25% level of the relevant segment.

After assigning firms into the 21 finer rating categories for the CRI sample over the 18-year sample period, we group all firms into the 9 consolidated rating categories, tally the results, and gross up by the other-exit rates to generate the model's implied 9 by 10 average realized migration matrix. Denote the model's final implied rating migration matrix by $\boldsymbol{M}(\boldsymbol{\theta})$.

The Calibration Objective

The PDiR2.0 model calibration is to find $\boldsymbol{\theta}$ to minimize the sum of squared differences between two rating migration matrices, i.e., $\boldsymbol{M}(\boldsymbol{\theta})$ and $\widehat{\boldsymbol{M}}$. However, we include only three components in the calibration target, and they are the diagonal, two immediate off-diagonal terms (one in each direction) and the 10th column holding default rates. Other elements of the 9 by 10 matrix are ignored because their values are small and prone to sampling errors. In addition to the constraint of increasing values placed on the elements of $\boldsymbol{\theta}$, we require the proportion of AAA firms in the CRI sample to be no less than 1.5%, a

level comparable to that of the S&P global corporate rating pool. Without it, we would have substantially fewer AAA firms.

The minimization is executed by adopting the density-tempered sequential Monte Carlo (SMC) technique of Duan and Fulop (2015), which also serves as the optimizer for the $PDiR_{old}$ method. The optimization algorithm is described in Duan and Li (2021).

Results

We take the default column out of the migration matrix (which is grossed-up by the other-exit rates) to perform a comparison between the realized default rates of the rating agencies and those produced by PDiR2.0. The comparison indicates that the realized default rates from the rating agencies and PDiR2.0 are reasonably close. Worth noting is the noticeable differences between S&P's and Moody's realized default rates. Naturally, they will result in different PDiR2.0 boundary values.

Table 2. Mapping of Default Rates (Grossed-up by Exit Rates)

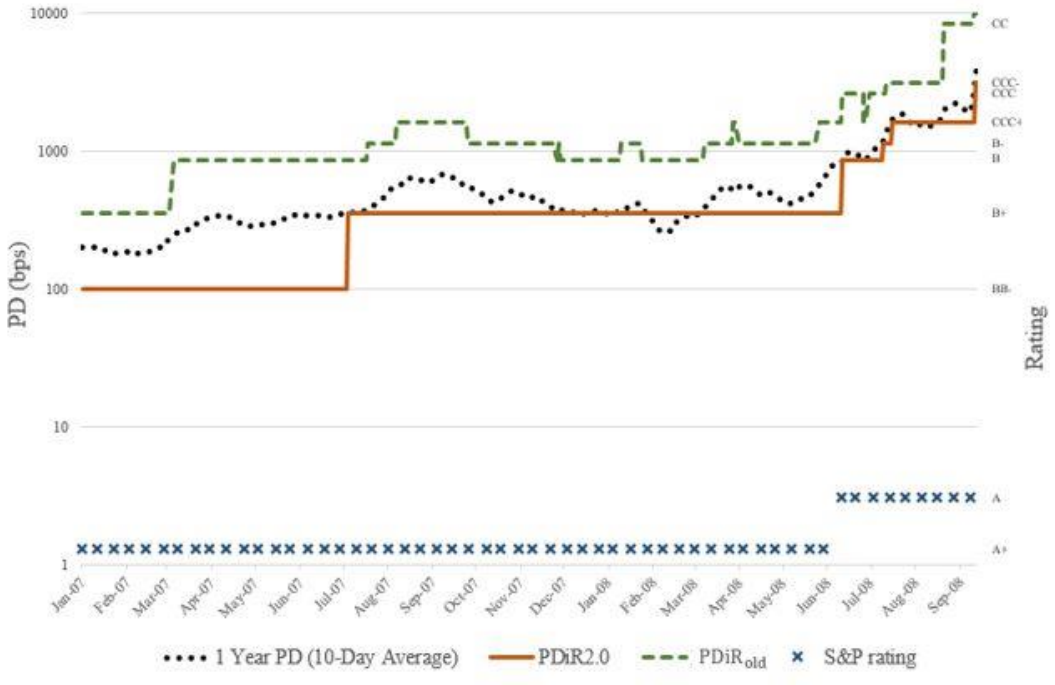
	S&P reported	PDiR2.0 realized		Moody's reported	PDiR2.0 realized
AAA	0.0000%	0.0552%	**Aaa**	0.0000%	0.0203%
AA	0.0147%	0.0282%	**Aa**	0.0490%	0.0278%
A	0.0516%	0.0401%	**A**	0.0860%	0.0388%
BBB	0.1676%	0.1324%	**Baa**	0.1972%	0.1274%
BB	0.7479%	0.7965%	**Ba**	0.6375%	0.4679%
B	3.7477%	4.6090%	**B**	2.9904%	1.9009%
CCC	27.6436%	26.8206%	**Caa**	11.5902%	5.9122%
CC	50.5455%	56.5957%	**Ca**	27.3585%	17.2645%
C	50.0000%	66.6667%	**C**	29.6875%	37.0262%

IV. AN ILLUSTRATIVE EXAMPLE

We use Lehman Brothers Holdings for an illustration of PDiR2.0 vis-à-vis $PDiR_{old}$. Lehman Brothers filed for a US Chapter 11 bankruptcy in September 2008, setting off a global financial crisis. Surprisingly perhaps, all major commercial credit ratings on Lehman Brothers virtually stayed unchanged in the three years leading to its bankruptcy. Evidently, the S&P ratings in Figure 1 started at A+ and was only downgraded to A in mid-2008, just a couple of months prior to its spectacular collapse.

The figure reveals that both PDiR2.0 and $PDiR_{old}$ ratings are much more responsive to Lehman Brothers' deteriorating credit quality as compared with the S&P credit rating. PDiR2.0 exhibits more rating stickiness, reflective of a design feature. PDiR2.0 also assigns better ratings as compared to $PDiR_{old}$. An early warning signal from PDiR2.0 or $PDiR_{old}$ is evident in Figure 1 where the implied rating started below the investment grade in January 2007 and worsened steadily to B and lower. This early warning feature is rooted in the responsive CRI-PD which reacts to its deteriorating credit quality.

Figure 1. Lehman Brothers' one-year PDs & ratings



V. CONCLUSION

The PDiR2.0 complements the CRI-PD by providing a quick and intuitive indicator on the credit quality of firms through referencing a rating agency's rating scale and migration pattern. It is an enhanced version in the sense that the new methodology no longer requires to proxy default rates for, say, the S&P AA+ and AAA categories, which are to some extent derived from an arbitrary linear extrapolation. The inbuilt rating stickiness of PDiR2.0 is also a useful feature because it resembles the practice of credit rating agencies. Furthermore, the resulting overall PDiR2.0 rating distribution of the CRI sample is closer to that of the global corporate rating pool to which it references.

This paper provides in the beginning the mapping table calibrated to the migration experience of the S&P global corporate rating pool over an 18-year period (2000-2017). A similar mapping table corresponding to Moody's over 19 years (2000-2018) is given at the end. The PDiR2.0 methodology is obviously applicable to other large rating pools, be it a pool of large corporates, SMEs or consumers.

VI. REFERENCES

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Table 3. Mapping 10-day moving average 1-year CRI PDs to Moody's experience

Rating Category	initial assignment		Upgrade to		Downgrade to	
	lb (bps)	ub (bps)	lb (bps)	ub (bps)	lb (bps)	ub (bps)
Aaa	0	0.0065	0	0.0049	-	-
Aa1	0.0065	0.0662	0.0049	0.0065	0.0662	0.1860
Aa2	0.0662	0.1860	0.0065	0.0662	0.1860	0.2450
Aa3	0.1860	0.2450	0.0662	0.1860	0.2450	0.8970
A1	0.2450	0.8970	0.1860	0.2450	0.8970	2.2008
A2	0.8970	2.2008	0.2450	0.8970	2.2008	2.8526
A3	2.2008	2.8526	0.8970	2.2008	2.8526	9.2961
Baa1	2.8526	9.2961	2.2008	2.8526	9.2961	22.1830
Baa2	9.2961	22.1830	2.8526	9.2961	22.1830	28.6265
Baa3	22.1830	28.6265	9.2961	22.1830	28.6265	43.3585
Ba1	28.6265	43.3585	22.1830	28.6265	43.3585	72.8224
Ba2	43.3585	72.8224	28.6265	43.3585	72.8224	87.5544
Ba3	72.8224	87.5544	43.3585	72.8224	87.5544	122.7975
B1	87.5544	122.7975	72.8224	87.5544	122.7975	193.2836
B2	122.7975	193.2836	87.5544	122.7975	193.2836	228.5267
B3	193.2836	228.5267	122.7975	193.2836	228.5267	356.9571
Caa1	228.5267	356.9571	193.2836	228.5267	356.9571	613.8180
Caa2	356.9571	613.8180	228.5267	356.9571	613.8180	742.2484
Caa3	613.8180	742.2484	356.9571	613.8180	742.2484	857.5755
Ca	742.2484	1203.5566	613.8180	1088.2295	857.5755	3402.6674
C	1203.5566	10000	-	-	3402.6674	10000

ABOUT THE CREDIT RESEARCH INITIATIVE

The Credit Research Initiative (CRI) was launched by Professor Jin-Chuan Duan in July 2009 at the Risk Management Institute of the National University of Singapore. Aiming at "Transforming Big Data into Smart Data", CRI covers over 85,000 public firms and produces daily updated Probabilities of Default (1-month to 5-year horizon), Actuarial Spreads (1-year to 5-year contract) and Probability of Default implied Ratings on over 43,000 currently active, exchange-listed firms in 133 economies. CRI also distributes historical time series of over 42,000 inactive firms due to bankruptcy, corporate consolidation or delisting for other reasons. In addition, CRI produces and maintains Corporate Vulnerability Indices (CVI), which can be viewed as stress indicators, measuring credit risk in economies, regions, and special portfolios.

As a further step, CRI converts smart data to actionable data to offer bespoke solutions to meet demands of its users. A concrete example is our development of the BuDA (Bottom-up Default Analysis) toolkit in collaboration with the International Monetary Fund (IMF). BuDA is an automated analytic tool based on the CRI-PD system, enabling IMF economists to conduct scenario analyses on the macroeconomic and financial linkage.

CRI publishes Weekly Credit Brief and Semi-Annual Credit Summary, highlighting key credit-related events, offering insights based on the CRI-PDs of the entities involved, and providing useful statistics on credit risk of economies and/or sectors.

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